PRICING ELECTRICITY OPTIONS USING STOCHASTIC OPTIMIZATION

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III CONGRESO COLOMBIANO DE INVESTIGACIÓN DE OPERACIONES
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JESÚS MARÍA VELÁSQUEZ BERMÚDEZ
jesus.velasquez@decisionware.net

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ABSTRACT

In an open electricity market the decision maker, buyer or seller, must face at least two risks: the price risk and the volumetric risk. Based in an stochastic optimization model of electricity trading multiples effects in pricing of electricity options are studied. The effects are: demand level, generation level, load curve, contracts and alternative bids. Additionally, the study considers the effect of three possibilities to determine the objective function: maximize the expected incomes, maximize the minimal incomes, and expected incomes and conditional Value-at-Risk incomes constraints. Numerical cases are presented. The results demonstrate that the conventional methodologies of pricing options, based only on the probabilistic characteristics of the future spot price, ignore important aspects that affect the optimum value of the options.

1.  INTRODUCTION

In the competitive electrical markets is increasing the incorporation of standard financial instruments for the market risk hedging (price and volume) that the agents should face. One of the instruments is options to supply in the future electricity blocks. An option gives the holder the right to buy/sell electricity blocks at a specified exercise price, at some time in the future. Unlike a forward or futures contract, an option contract does not oblige the holder to buy/sell the electricity blocks. The price of an option contract is comprised of two elements: i) strike price: the price paid if the option is exercised when the holder “calls” for the contract to be fulfilled, ii) option fee (premium): the sum of money paid by the holder (buyer) of an option contract and received by the issuer (seller).
The seminal work of Black and Sholes (1973) was the starting point to establish the premium of an option. Taking into account the peculiarities of the electrical markets and of the electrical systems, it has been necessary to developed studies in which has been proposed to modify the Black and Sholes model (1973) to value this type of instrument (Clewlow, L. and Strickland 1999, Zeng, L. 2000). Also methodologies based on "Montecarlo" simulation are found as an alternate mechanism to value the options (Joy et al., 1996).

The previous models, oriented to estimate the option fee in the market, have as fundamental hypothesis that the value of the option is function exclusively of the probabilistic characteristics of the future spot price and the strike price. The previous hypothesis is valid when the agent that negotiates the option is not affected by an specific decisions environment that differentiate it from other agents. The "classic" methodologies ignore the effect that the decisions environment has in the valuation of the option fee. Some of the aspects of this environment are the demand that it must attend (variability, magnitude and load curve), the contracts of purchase/sale that it has established, the alternative bids that exists in the market, and the generation assets structure. All the previous factors affect the opportunity values that the agent has and therefore affect the option fee that the agent is disposed to paying and/or to collect.

The stochastic optimization models including synthetic scenarios are appropriate to consider explicitly the agent decisions environment (Ahn et. al. 1994, Birger et al. 2001, Pereira et al. 2000, Velásquez 2001a-2001b,). In these models the instrumental variables are the amount to trade for each financial instrument in the first stage of the planning horizon. The dual variables and the reduced costs (simplex multipliers) provide information with respect to the marginal benefits/costs associated with each type of transaction that is considered in the model and may be used to calculate equilibrium prices for the different risk hedging instruments. The following sections present the use of a stochastic optimization model to determine the option fee as function of the strike price and the agent decisions environment.

Additionally, to the effects due to the decisions environment, it should be take into account the effect due to the agent position with respect to the risk. This position is represented by the optimization objective function and by utility function that the agent select to evaluate its decisions, this effect also is analyzed, including the incorporation of constraints about limits in the Value-at-Risk (VaR) of the agent portfolio.

2. REFERENCE FRAMEWORK

2.1. MATHEMATICAL MODELING

The model formulated by Velasquez (2001a, 2001b) is taken as reference; the Annex A presented a resume of the model that permits to analyze a great variety of cases of negotiation (bilateral contracts, forwards, options) and it can be used to analyze long term
(years-months) and/or short term (months-days) periods. All the accomplished experiments are associated with long term decisions.

2.2. EQUILIBRIUM PRICES

To determine the equilibrium prices for different negotiation modalities it should be take into account the reduced costs of the variable that represent the magnitude negotiated in each type of transaction. Below the methodology to follow in the case of a model that minimize the trading cost is described. For each type of transaction, index i, the following definitions are considered:

- \( X_i \): Quantity negotiated
- \( CR_i \): Unitary reduced costs (simplex multiplier)
- \( L_i \): Lower bound for \( X_i \)
- \( U_i \): Upper bound for \( X_i \)
- \( C_i \): Unitary cost
- \( P_i \): Equilibrium price

The values of \( X_i \) and \( CR_i \) are results of the optimization process. \( L_i, U_i, \) and \( C_i \) correspond to parameters of the optimization model. The value of \( P_i \) is calculated as:

\[
P_i = C_i - CR_i
\]

It should be take into account that in the optimum solution \( CR_i \) complies with

\[
0 \leq CR_i \quad \text{Si} \quad X_i = L_i
\]
\[
0 = C_i \quad \text{Si} \quad L_i < X_i < U_i
\]
\[
CR_i \leq 0 \quad \text{Si} \quad X_i = U_i
\]

The equilibrium prices are valid in the neighborhood of the optimum solution and for differential changes in the value of the parameters. To extend the validity of the prices to distant regions is necessary to accomplish parametric analysis of the environment.

To determine the premium for the options \( i \) we consider the following definitions:

- \( O_i \): Quantity negotiated
- \( PS_i \): Strike price
- \( PR_i \): Reference option fee
- \( PO_i \): Equilibrium option fee
- \( CRR_i \): Reduced cost for the reference option fee
- \( \varepsilon \): Size of the reference neighborhood

To maintain the validity of the results \( O_i \) it should be restrict to:

\[
0 < O_i < \varepsilon
\]

The "optimum" option fee for the option type \( i \) is
\[ PO_i = PR_i - CRR_i \]

For convenience the reference option fee is assumed equal to zero \((PR_i=0)\). Taking into account the relationship between \(O_i\) and \(PS_i\) the "optimum" option fee as function of the strike price \(PS_i\) is equal to

\[ PO_i = -CRR_i = f(PS_i) \]

2.3. **MODELING OPTIONS**

For simplicity the analysis only presents the case of "call" options (purchase of electricity) pointing out that the case of "put" options is similar. The option modalities considered in the model are monthly options of annual blocks and monthly options of monthly blocks. For a year of planning we consider the following definitions:

- \(PSP_{t,h,r}\) Spot price spot in the hour \(h\) of the month \(t\) in the scenario \(r\)
- \(PSA_i\) Strike price for annual options type \(i\)
- \(PSM_{i,t}\) Strike price for the month \(t\) for monthly options type \(i\)
- \(PEA_{i,r}\) Effective price of the annual option type \(i\) in the scenario \(r\)
- \(PEM_{i,t,r}\) Effective price in the month \(t\) of the monthly option type \(i\) in the scenario \(r\)
- \(NH_t\) number of hours of the month \(t\)

The effective price is referred to real cost that it must pay the buyer of the option taking into account the relationship between the strike price and the spot price. It corresponds to a random variable that is defined depending on the type on option and of the form on exercising the option. For the case of annual block options exists a strike price yearlong, for the monthly block options exists a strike price for each month of the planning horizon. All options are exercised hour by hour.

The net income due to the exercise or not of the options are random, depends on the spot price and it can be expressed as the product of the effective price by the negotiated options quantity. Below it is presented the determination of the effective price for each type of option:

- Annual options for all the hours of the planning horizon
  \[ PEA_{i,r} = \sum_t \sum_h \text{Minimum} ( PSA_i , PSP_{t,h,r} ) \]
- Monthly options for all the hours of each month of the planning horizon
  \[ PEM_{i,t,r} = \sum_h \text{Minimum} ( PSM_{i,t} , PSP_{t,h,r} ) \]

3. **CONTROLLED EXPERIMENTS**

Below the analysis accomplished to determine the effect of different factors of the agent decisions environment is presented. The planning horizon is a year. The experiments take
as reference realistic data of the Colombian electrical market and study the following sources of variation:
  ▪ Volume of demand  
  ▪ Assets generation  
  ▪ Load Curve  
  ▪ Contracts  
  ▪ Alternative bids  
  ▪ Objective Functions  
  ▪ VaR constraints

The analysis is presented in four sections. In the first and in the second sections are studied the firsts five sources of variation fixing as optimization criterion to minimize the maximum cost. Two cases are considered: a pure distributor without generation that it must attend an own and/or a contracted demand, and a company that integrate two electrical businesses: distribution and generation. The third section compares two optimization criteria: to minimize the expected cost and to minimize the maximum cost. Finally, it is accomplished the risk analysis incorporating Value-at-Risk constraints and minimize the expected cost.

For each type of agent has been built a reference case with respect to which are presented the effects of the different factors. The information about the spot price volatility is equal for all cases and is synthesized in 10 synthetic scenarios that are presented in the following graph, the bulk line corresponds to the average value for each month. The annual average spot price is 38731 $/MWh (Colombian pesos by megawatt-hour).

3.1. DISTRIBUTOR

This section is related with an electricity distributor, or a trader, that it does not have plants for electricity generation and must attend its regulated demand and/or its contracted demand.

3.1.1. REFERENCE CASE

In the reference case the demand is considered deterministic with only one typical day. Additionally the aggregate demand for each month of the planning horizon is known. The
hourly demand is calculated combining the load curve with the projection of aggregate monthly demand. The aggregate annual demand is 51778 MWh.

The curves of the option fee versus the strike price are presented below. Initially the case of a yearlong option and below the curves for monthly options.

3.1.2. CONTRACTS

The first factor analyzed is the contracts of purchase of electricity that the agent may have subscribed to attend its demand, that implies smaller exposition to the spot market risk. It was considered two contracts with blocks of yearlong constant electricity of 120 MWh and 240 MWh. The results are presented below:
The results show that there is a light decrease effect in the option fee due to level of the contracts. The effect is meaningful in January and in February. The sense of the change is logical and reflects that a greater level of contracting implies smaller requirements of risk hedging and therefore smaller availability to pay for the options.

3.1.3. LOAD CURVE

Below the effects due to the form of the load curve are presented. To accomplish the analysis the load curve was distorted in a such way of changing its form maintaining constant the quantity of energy. Two extreme curves were considered: a constant flat load during all the day, and a concentrate load in the peak hours. The results are presented below:
The results show that exists a light effect in the option fee due to the form of the load curve. The effect is meaningful in January and in February. Seemed be that the load concentration benefits to the agent, the reason of this effect is not evident.

3.1.4. ALTERNATIVE OFFERS

This case considers the effect due to that the agent has the possibility to obtaining offers from other agents and that they can have a different perception of the spot market in the future. It is considered a hypothetical case in which the agent has received eleven offers in seven modalities. The numerical data are omitted, but are found in Velasquez (2001a). The price average of the offers is near 40000 $/MWh, this implies that the perception of the sellers is that the future spot price will be less than the perception of the buyer agent. The results are presented below.

The results show that exists a decrease effect in the option fee due to the quotes that receives the agent. The sense of the effect is logical, since the sellers agents have a smaller future spot prices perception that the buyer agent can capitalize.

3.1.5. RANDOM DEMAND

The following cases have as purpose to determine the dependency between the options fees and the demand variability. For the foregoing were generated demand synthetic series that have the same average that deterministic demand considered in the previous cases. Two series of demand with different variability was considered. The synthetic series are presented below, the bulk line corresponds to the average value.

The results are presented below
The results permit to conclude that the option fee depends of the variability of the demand. The results are logical, since when the variability decrease the option fee tends to be equal to the option fee calculates with deterministic demand, the reason why the option fee goes down when the variability goes up, it is not explained.

3.2. INTEGRATED AGENT

This section studies the effects in an integrated agent that it must attend a demand and that it has electrical generation plants.

3.1.1. REFERENCE CASE

The hypothesis with respect to the generation should take into account the regulation of the electrical market in which act the agent. In the Colombian case, the regulation does not give rights of auto-dispatch to the generators and therefore the physical dispatch can not be optimized jointly with the financial dispatch (contracts and instruments); this condition may be change if the joint dispatch is possible. For the foregoing the generation levels are considered exogenous to the model and correspond to parameters associated with the random scenarios. Ten synthetic series of generation have been considered, they are correlated with the synthetic series of spot prices. The scenarios, spot price and generation, were accomplished with a minimal cost electricity dispatch model (Velásquez and Nieto 1999).

The demand is the same used in the case of the pure distributor. The agent plant is called CHIVOR and its annual average generation are 3749 GWh, varying in a range between
2959 and 4780 GWh. The graphic presents the synthetic series of generation, the bulk line corresponds to the average.

3.2.2. BUSINESS INTEGRATION

Below it is analyzed the effect related to the businesses integration: distribution (trading) and generation. The following graph compares the option fees in the two references cases.

The results are coherent with the logic, the integrated planning implies that the option fee is bottommost due to the synergy generated by the joint decisionmaking. In the integrated case exists a clear differentiation between the prices of the monthly options for each month, this due to the seasonal variability of the water inflows. The next graphs presents the results.
3.2.3. DEMAND LEVEL

This case shows the effects due to variations in the demand level, the demand was parameterized based on a proportional factor of the reference demand. Below they are presented the results for a factor equal to 0.0 (no demand), 0.5 and 2.0 times the reference demand.

The results are coherent with the logic. When the level of demand increases, the options fee increases. However, the increase is not constant and tends to be stabilized. When there is no demand, the value of the "call" options is zero, since the agent must act as seller and not as buyer.

3.2.4. OFFERS
This case considers the effect due to the possibility that has the agent obtains quotes from other agents. The same offers that in the case of the distributor are considered. The results are presented below.

The results show that a real effect in the option fee does not exist due to the quotes that receive the integrated agent. Departing of the case of the pure distributor the effect due to the generation is greater than the effect due to the alternative offers, and by this reason the offers lose its opportunity value when the agent has its own generation.

3.2.5. GENERATION PLANTS

This case studies the effect of different types of generation plants. Two plants are considered: BETANIA a hydroelectric with low regulation capacity, with an annual average generation of 2316 GWh, varying in a range between 1829 and 3036 GWh, and FLORES1 an efficient thermoelectric with an annual average generation of 1030 GWh, varying in a range between 861 and 1116 GWh, depending of the hydrologic conditions. The synthetic series of generation are presented below.

The results are presented below.
The results show that exists an effect in the option fee as consequence of the characteristics of the type of plant. It can be noted that for the hydraulic plants the options fee is equal in both cases; this it should be, perhaps, to the generation overflow with relation to demand. On the other hand, in the case of the thermal plant the option fee tends to be equal to the case of the pure distributor, this may indicate that the efficiency of the plant and the generation capacity does not provide real advantages to reduce the value of the option for the agent.

Observing the foregoing, it can be concluded that to establish the option fee in some cases is more important the generation, and in other the demand, this implies that the value of the option depends on the assets structure of the agent. However, the option fee seems to be liberated of the numerical values, reflecting the negotiation position and the perception of the future spot price. Depending on the balance generation-demand should exist cases in which the value of the options is in intermediate points.

3.3. OBJECTIVE FUNCTIONS

These cases was developed to determine the capacity of the objective function used in the stochastic optimization model to detect the effects derived from the agent decisions environment. Two objective functions, extremes with respect to the attitudes to the risk, were compared: the expected value of the cost (extremely risk taker) and the maximum cost (extremely risk averse). Two cases of the previously presented were analyzed: the case of reference and the case with alternative offers, each case was analyzed for the distributor and for the integrated agent.
The main result of the experiments is that the objective function associated for expected value of the costs can not distinguish the effects due to different decisions environments. The solution is indifferent to any variation in the factors of the environment. This means that the value of option only depends on the probabilistic perception about the future spot price, being similar to the results of the Black&Sholes and its variations models, that only consider this information to determine the option fee. The graphical results are showed below, it exists only one graphic for the four cases.

The following graphics present the comparative analysis of the two objective functions.

3.4. RISK ANALYSIS

This section analyzes the decision-making process when a stochastic optimization model that minimizes expected cost and includes restrictions on the risk level using the CVaR (Conditional Value-at-Risk index) (Ursayev 2000) as risk measure is used. The case of the distributor with alternative offers was take as reference because the variations in the option fee are noted with greater facility. Below experiments with several limits for the CVaR fixing the probability level in 0.90 are presented.

### COMPARATIVE ANALYSIS WITH PROBABILITY LEVEL FIXED

<table>
<thead>
<tr>
<th>TOTAL COST ($**6)</th>
<th>MINI MAX</th>
<th>$R = 0.90 - CVaR LIMIT ($**6)</th>
<th>EXPECTED VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>53000</td>
<td>54000</td>
</tr>
<tr>
<td>MEAN</td>
<td>52402</td>
<td>50305</td>
<td>49822</td>
</tr>
</tbody>
</table>
The results are coherent with the theory, when the CVaR limit is reduced the solution tends to the results of minimization of the maximum cost, when the limit is increased the solution tends to the expected cost solution. The following graph presents the six constrained cases plus the case of CVaR limit equal to infinite, that coincides with the unconstrained risk minimization of the expected cost.

Below the value of the monthly options for the two extreme cases, CVaR $\leq 53000$ and CVaR $\leq 58000$ are presented.

It is clear that the options fee depends on the risk level that wish to assume the agent.

4. CONCLUSIONS

It can be concluded that the optimum price of the options depends on the agent decisions environment. Therefore the valuation methods only based on the probabilistic characteristics of the future spot price ignore these aspects, and then produce results biased with respect to the “real” opportunity value that an option has for an agent. The stochastic optimization models are a good alternative to capture these effects and to provide optimum prices that take into account the opportunity costs/benefits that are due to the agent decisions environment.

The use of this of approach permits to learn about the most crucial factors than affect the option fee when they are valued as any asset than it must produce a real added value to the agent. Using as reference the values of the dual variables associated with physical and
financial resources of the trading stochastic optimization models is possible to obtain optimum options fees.

The use of stochastic optimization models that have the expected cost/benefit as objective function can not distinguish the effects of agent decisions environment and its results are equal to the obtained using methodologies that only consider the probabilistic characteristics of the future spot price. In this way the value added that would produce a solid methodology like the stochastic optimization is lost. The incorporation of CVaR constraints gives sense to the optimization based on the expected value.

REFERENCES


ANNEX A

OPTMER: A MODEL TO OPTIMIZE THE DECISIONS OF ELECTRICITY TRADING
The modeling process must consider that the regional regulatory entities have different conceptions about what "must be" a competitive electricity market. Therefore, aspects of the models that depend on each specific market exist. This model assumes a "simple" market in which the agents can accomplish long term transactions under different modalities and short term transactions in the spot market.

The model assumes that exist synthetic series of spot prices that represent the random environment of the decision making process. Each synthetic series is associated to a possible future scenario in a stochastic optimization model called as OPTMER (in Spanish "OPTmización del MERcadeo") that it is a stochastic linear optimization model that supports the decisions of an electricity generator, of a distributor, of a trader and/or of a vertical integrated company. This annex only consider the math formulation for a trader that accomplishes purchase/sale transactions with multiple agents or clients. In the spot market the trader purchases/sells the deficit/surpluses not covered in the long term market.

According to the typical characteristics of the prices structure in the electricity markets the modeling is based on the following concepts:

- Planning Periods: one, or several months
- Day Types: ordinary, Saturday and holidays.
- Load Blocks: a hour or a group of hours
- Contracts: Take or Pay and Options
- Negotiation Modalities:
  - free blocks: irregular electricity sales for any load block
  - monthly blocks: electricity blocks for all hours of a month, or a period.
  - annual blocks: electricity blocks for all hours of the year, or the planning horizon.
  - monthly blocks modulated by a load curve: electricity blocks for all hours of a month as a percentage of the buyer load curve
  - annual blocks modulated by a load curve: electricity blocks for all the hours of a year as a percentage of the buyer load curve
  - monthly options: options of electricity blocks for all hours of a month
  - annual options: options of electricity blocks for all hours of a year

To be short, the detailed math formulation is limited to present the parameters, the equations and the variables related to purchase offers and ignores the possibility of sale offers, those which can be included using a similar procedure to the described in the present annex.

### A.1. DEFINITIONS

The index used in the model are \( t \) for period (month), \( d \) for type of day, \( b \) for load block, \( g \) for agent that sells electricity and \( h \) for random scenario.

The parameters used in the model are (in cursive letter):

- Deterministic parameters
Electricity prices ($/MWh)

- **PCBA<sub>g</sub>** Agent <i>g</i> price for annual blocks
- **PCBM<sub>t,g</sub>** Agent <i>g</i> price for monthly blocks in month <i>t</i>
- **PCLI<sub>t,d,b,g</sub>** Agent <i>g</i> price for free blocks in month <i>t</i> type day <i>d</i> block <i>b</i>
- **PCMA<sub>g</sub>** Agent <i>g</i> price for modulate annual blocks
- **PCMM<sub>t,g</sub>** Agent <i>g</i> price for modulate monthly blocks in month <i>t</i>
- **PCOA<sub>g</sub>** Agent <i>g</i> price for annual options
- **PCS<sub>g</sub>** Agent <i>g</i> strike price for annual options
- **PCOM<sub>t,g</sub>** Agent <i>g</i> price for monthly options in month <i>t</i>
- **PCSA<sub>g</sub>** Agent <i>g</i> strike price for monthly options in month <i>t</i>

Electricity quantities (MWh)

- **DVBA<sub>g</sub>** Agent <i>g</i> availability for annual blocks
- **DVMM<sub>t,g</sub>** Agent <i>g</i> availability for modulate monthly blocks in month <i>t</i>
- **DVBM<sub>t,g</sub>** Agent <i>g</i> availability for monthly blocks in month <i>t</i>
- **DVMA<sub>g</sub>** Agent <i>g</i> availability for modulate annual blocks
- **DVLI<sub>t,d,b,g</sub>** Agent <i>g</i> availability for free blocks in month <i>t</i> type day <i>d</i> block <i>b</i>
- **DVOA<sub>g</sub>** Agent <i>g</i> availability for annual options
- **DVOM<sub>t,g</sub>** Agent <i>g</i> availability for monthly options

- **Random Parameters (components of the scenario vector Y<sub>h</sub>)**
  - **DEM<sub>t,d,b,h</sub>** Demand (regulated plus contracts) in the month <i>t</i> type day <i>d</i> block <i>b</i> under random condition <i>h</i>
  - **PSP<sub>t,d,b,h</sub>** Spot price in month <i>t</i> type day <i>d</i> block <i>b</i> under random condition <i>h</i>
  - **ψ<sub>t,h</sub>** Coefficient associated with the payment capacity of the spot market in month <i>t</i> random condition <i>h</i>
  - **ϕ<sub>t,h</sub>** Coefficient associated with payment time of the spot market in month <i>t</i> random condition <i>h</i>

The variables used in the model are (in normal letter):

- **Long term market decisions (deterministic variables, components of X)**
  - **CLP<sub>t,d,b,g</sub>** Total electricity bought to the agent <i>g</i> in month <i>t</i> type day <i>d</i> block <i>b</i>
  - **CBA<sub>g</sub>** Electricity bought in annual blocks to the agent <i>g</i>
  - **CBM<sub>t,g</sub>** Electricity bought in monthly blocks to the agent <i>g</i> in month <i>t</i>
  - **CLI<sub>t,d,b,g</sub>** Electricity bought in free blocks to the agent <i>g</i> in month <i>t</i> type day <i>d</i> block <i>b</i>
  - **CMA<sub>g</sub>** Fraction of electricity demand bought in annual modulate blocks to the agent <i>g</i>
  - **CMM<sub>t,g</sub>** Fraction of electricity demand bought in monthly modulate blocks to the agent <i>g</i> in month <i>t</i>
  - **COA<sub>g</sub>** Annual options of electricity blocks bought to the agent <i>g</i>
  - **COM<sub>t,g</sub>** Monthly options of electricity blocks bought to the agent <i>g</i> in month <i>t</i>

- **Simulated variables (random variables for each scenario)**
  - **VMS<sub>t,d,b,h</sub>** Sales in the spot market in month <i>t</i> type day <i>d</i> block <i>b</i> under random condition <i>h</i>
$CMS_{t,d,b,h}$ Purchases in the spot market in month $t$ type day $d$ block $b$ under random condition $h$

A.2. INCOME FUNCTION

The income is split into deterministic and stochastic. The deterministic part, $d(X)$, does not depend on the random conditions and corresponds to commitments derived from the decisions in the long-term market: the costs of blocks of electricity and the costs of the options. $d(X)$ can be expressed as

$$d(X) = \sum_{i} CBA_{i} + \sum_{g} COA_{g} + \sum_{t} \{ CBM_{t,c} + \sum_{i} PCBM_{t} + \sum_{i} COM_{t,i} + \sum_{i} PCOM_{t,i} \}$$

where $X$ represents the decision, vector related with long term market transactions. The stochastic income, $r(X|Y_{h})$, correspond to the purchases or sales in the spot market and to the exercise, or not, of the options. They depend on the random condition $h$, and can be expressed as

- **COP(X|Y_{h})**: expenditures by exercises the options
  $$COP(X|Y_{h}) = \sum_{g} \sum_{t} \sum_{d} \sum_{b} \text{Minimum}(PCSA_{g},PSP_{t,d,b,h}) \text{COA}_{g} + \text{Minimum}(PCSM_{t,g},PSP_{t,d,b,h}) \text{COM}_{t,g}$$

- **IMS(X|Y_{h})**: incomes by sales in the spot market
  $$IMS(X|Y_{h}) = \sum_{t} \sum_{d} \sum_{b} \text{Maximum}(0, \sum_{g} CLP_{t,d,b,g} - DEM_{t,d,b}) \text{PSP}_{t,d,b,h} \Psi_{t,h} \Phi_{t,h}$$

- **EMS(X|Y_{h})**: expenditures by purchases in the spot market
  $$EMS(X|Y_{h}) = \sum_{t} \sum_{d} \sum_{b} \text{Maximum}(0, DEM_{t,d,b} - \sum_{g} CLP_{t,d,b,g}) \text{PSP}_{t,d,b,h}$$

where $Y_{h}$ represents the random parameters vector associated with the condition $h$. The income and the expenditures in the spot market are considered independently due to the fact that would exist asymmetry in the spot market payment conditions ($\Psi_{t,h}$ and $\Phi_{t,h}$). The previous financial movements are caused in the future and represent the risk of the decision. Their net value is $r(X|Y_{h})$

$$r(X|Y_{h}) = IMS(X|Y_{h}) - COP(X|Y_{h}) - EMS(X|Y_{h})$$

The total income, deterministic plus stochastic, is

$$f(X|Y_{h}) = d(X) + r(X|Y_{h})$$
If a stochastic linear programming model is formulated, the variables contained within the Maximum function must be represented by a set of linear equations using the process that is described below. The following expression is considered

\[
\text{Maximum} \left[ 0, z \right] P
\]

(A:7)

where the variable \( z \) is not restricted and it is represented as the difference of two positive variables

\[
z = z^+ - z^-
\]

(A.8)

based on the previous change of variables we have

\[
\text{Maximum} \left[ 0, z \right] P = z^+ P
\]

(A.9)

For a correct representation, it should be to guarantee that one of the two \( z \)-variables will be equal to zero, what is procured in linear programming due to the colineality between \( z^+ \) and \( z^- \). Based on the foregoing, the following definitions can be considered

\[
z_{t,d,b,h} = VMS_{t,d,b,h} - CMS_{t,d,b,h} = \sum_g CLP_{t,d,b,g} - DEM_{t,d,b}
\]

(A.10)

where \( CLP_{t,d,b,g} \) represents the total purchases of electricity to the agent \( g \) in month \( t \) type day \( d \) block \( b \) under random condition \( h \) and it is defined by the sum of negotiation modalities

\[
CLP_{t,d,b,g} = CBA_g + CBM_t,g + COA_g + COM_{t,g} + CLI_{t,d,b,g} + CMM_{t,g} DEM_{t,d,b} + CMA_g DEM_{t,d,b}
\]

(A.11)

Then

\[
\text{ISM}(X|Y_h) = \sum_t \sum_d \sum_b VMS_{t,d,b,h} PSP_{t,d,b,h} \Psi_{t,h} \Phi_{t,h}
\]

(A.12)

\[
\text{ESM}(X|Y_h) = \sum_t \sum_d \sum_b CMS_{t,d,b,h} PSP_{t,d,b,h}
\]

(A.13)

The equations set \{A.1, A.2, A.5, A.6, A.10, A.11, A.12, A.13\} constitute a linear system that describes the income/expenditures that will have the agent and should make part of the optimization model; it will be called "the marketing process constraints".

A.3. OTHER CONSTRAINTS

A.3.1. LOAD MODULATION

The variables related to blocks modulated by a load curve are demand fractions and they should be ranged between 0 and 1

\[
0 \leq \text{CMM}_{t,g} \leq 1 \quad \text{(A.14a)}
\]

\[
0 \leq \text{CMA}_g \leq 1 \quad \text{(A.14b)}
\]
A.3.2. AVAILABILITY TO SALE

Normally, the agents receive electricity offers in those which a price is associated to a quantity that the seller is prepared to commit. This implies bounds for the quantities to buy

\[ CBA_g \leq DVBA_g \] (A.15a)
\[ CBM_{t,g} \leq DVBM_{t,g} \] (A.15b)
\[ CMA_g \leq DVMA_g \] (A.15c)
\[ CMM_{t,g} \leq DVMM_{t,g} \] (A.15d)
\[ COA_g \leq DVOA_g \] (A.15e)
\[ COM_{t,g} \leq DVOM_{t,g} \] (A.15f)
\[ CLI_{t,d,b,g} \leq DVLI_{t,d,b,g} \] (A.15g)