TUTORIAL – SESSION 2
IMPLEMENTATION OF THE VRP PROBLEM
(VEHICLE ROUTING PROBLEM)

SESSION 2: VRP MODELING IN EXCEL

Mathematical Modeling System

BASIC TUTORIAL

1. **SESSION 1: INTRODUCTION**
   - Introduction to OPTEX (Section 1)
   - OPTEX-EXCEL-MMS (Section 2)

2. **SESSION 2: VRP MODELING IN EXCEL**
   - VRP: Vehicle Routing Problem (Section 3)
   - Implementing VRP Model using EXCEL (Section 4)

3. **SESSION 3: USING EXCEL TO LOAD DATA**
   - Industrial Data Information Systems –IDIS- (Section 5)

4. **SESSION 4: OPTEX-GUI – LOADING MODELS**
   - Loading the Model in OPTEX-MMIS (Section 6)
   - Verification of the Model in OPTEX-MMIS (Section 7)

5. **SESSION 5: Loading and Checking Industrial Data**
   - Implementation and Validation of IDIS- (Section 8)

6. **SESSION 6: Solving Mathematical Models**
   - Scenarios and Families of Scenarios (Section 9)
   - Solution of Mathematical Problems (Section 10)
   - Results Information System (Section 11)

7. **SESSION 7: SQL Servers**
   - Using SQL Servers for IDIS (Section 12)

8. **SESSION 8: Optimization Technologies**
   - Solving Problems using C (Section 13.1)
   - Solving Problems using GAMS (Section 13.2)
   - Solving Problems using IBM OPL (Section 13.3)

Analytics

TUTORIAL

IMPLEMENTATION OF THE VRP PROBLEM

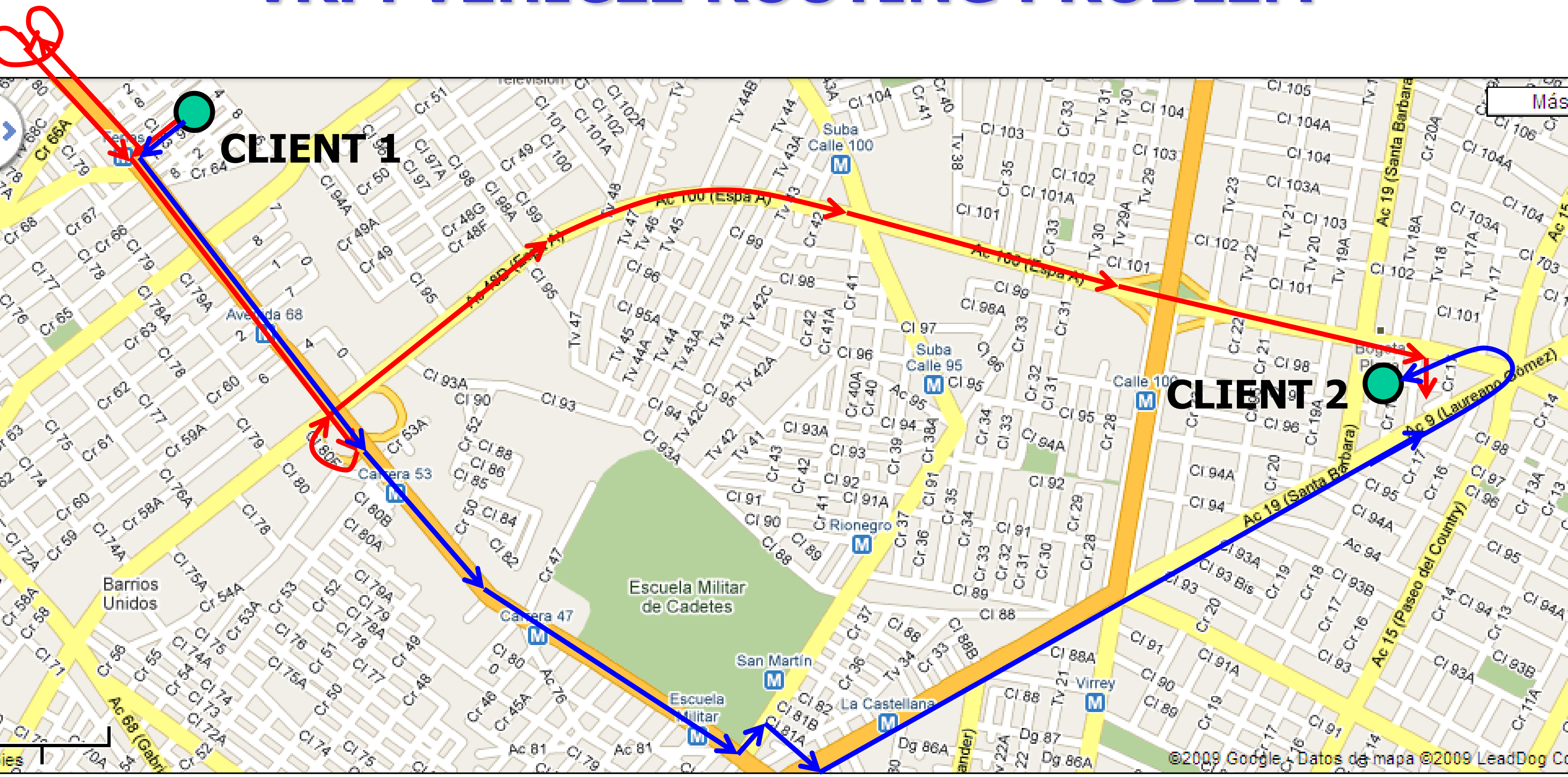
(VEHICLE ROUTING PROBLEM)

BASIC TUTORIAL

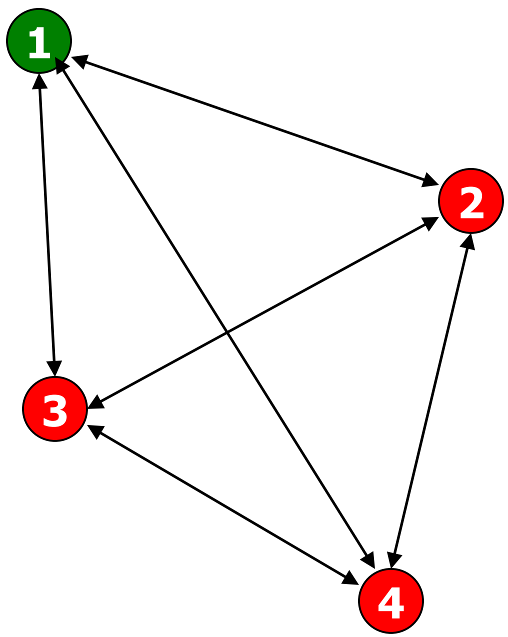
2. SESSION 2: VRP MODELING IN EXCEL

- VRP: Vehicle Routing Problem (Section 2)
- Implementing VRP Model using EXCEL (Section 4)

TSP: TRAVEL SALESMAN PROBLEM
VRP: VEHICLE ROUTING PROBLEM

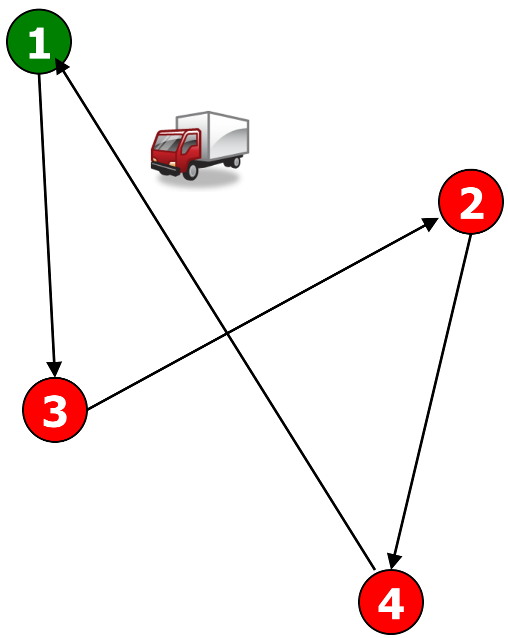


TSP: TRAVEL SALESMAN PROBLEM



Choose the optimal sequence that minimizes the costs of visiting all the nodes that make up a path, starting from a default source

TSP: TRAVEL SALESMAN PROBLEM



$$\textit{Min} \sum_i \sum_j c_{ij} x_{ij}$$

subject to

$$\sum_j x_{ij} = 1 \quad \forall i$$

$$\sum_j x_{ji} = 1 \quad \forall i$$

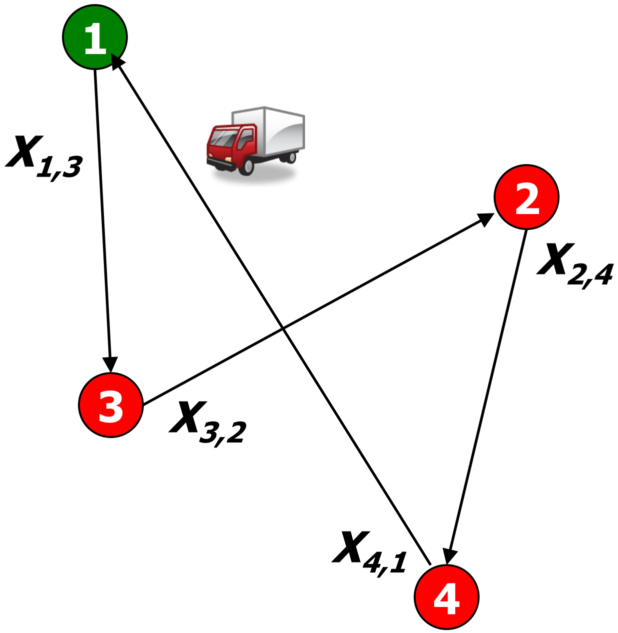
$$x_{ij} \in \{0,1\}$$

$$x_{ii} = 0$$

c_{ij} ***cost of going from i to j***

x_{ij} ***decision of going from i to j***

TSP: TRAVEL SALESMAN PROBLEM



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ij}$$

subject to

$$\Sigma_j\, x_{ij} = 1 \quad \forall i$$

$$\Sigma_j\, x_{ji} = 1 \quad \forall i$$

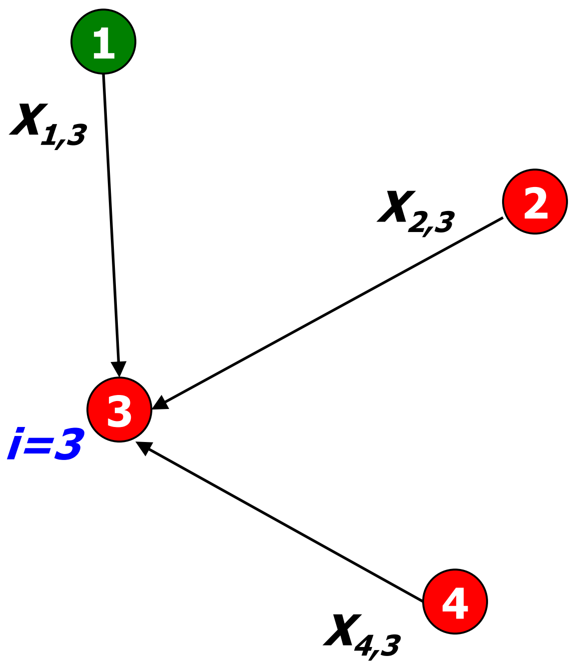
$$x_{ij} \in \{0,1\}$$

$$x_{ii} = 0$$

c_{ij} *cost of going from i to j*

x_{ij} *decision of going from i to j*

TSP: TRAVEL SALESMAN PROBLEM



Input balance equation

$$\text{Min } \Sigma_i \Sigma_j \; c_{ij} \, x_{ij}$$

subject to

$$\Sigma_j \, x_{ij} = 1 \quad \forall i$$

$$\Sigma_j \, x_{ji} = 1 \quad \forall i$$

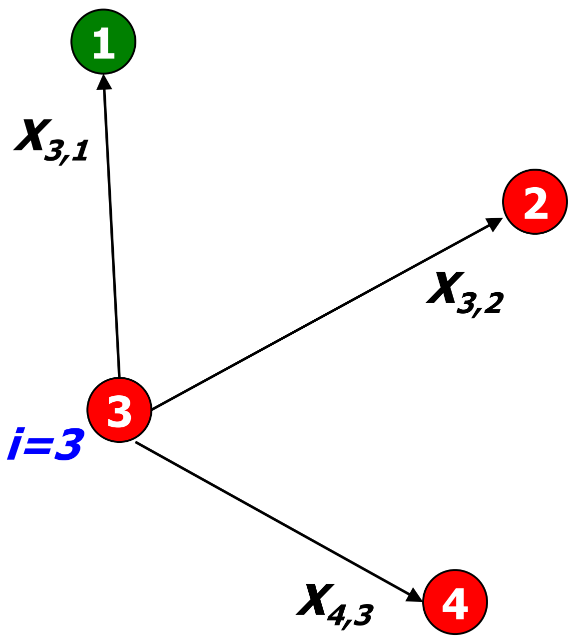
$$x_{ij} \in \{0,1\}$$

$$x_{ii} = 0$$

c_{ij} cost of going from i to j

x_{ij} decision of going from i to j

TSP: TRAVEL SALESMAN PROBLEM



Output balance equation

$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ij}$$

subject to

$$\Sigma_j\, x_{ij} = 1 \quad \forall i$$

$$\Sigma_j\, x_{ji} = 1 \quad \forall i$$

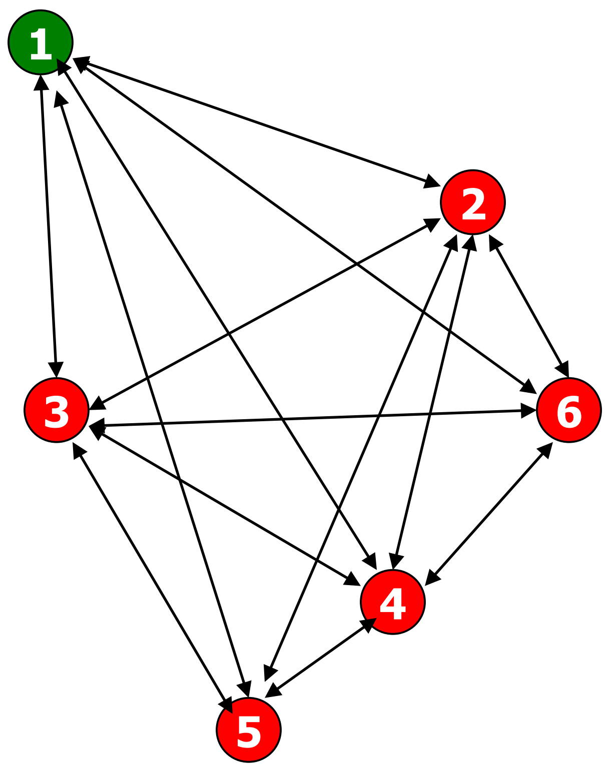
$$x_{ij} \in \{0,1\}$$

$$x_{ii} = 0$$

c_{ij} ***cost of going from i to j***

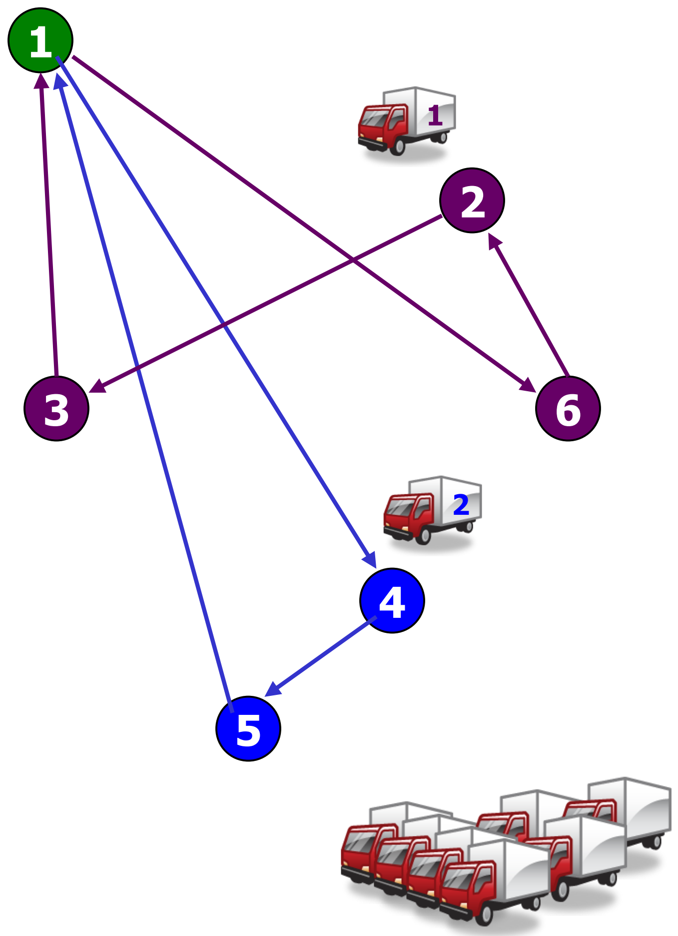
x_{ij} ***decision of going from i to j***

VRP: VEHICLE ROUTING PROBLEM



The problem is to determine the nodes that must integrate the different routes that minimize the costs of visiting all the nodes of a distribution/recollection system, starting from a default source , using a fleet of homogenous vehicles.

VRP: VEHICLE ROUTING PROBLEM

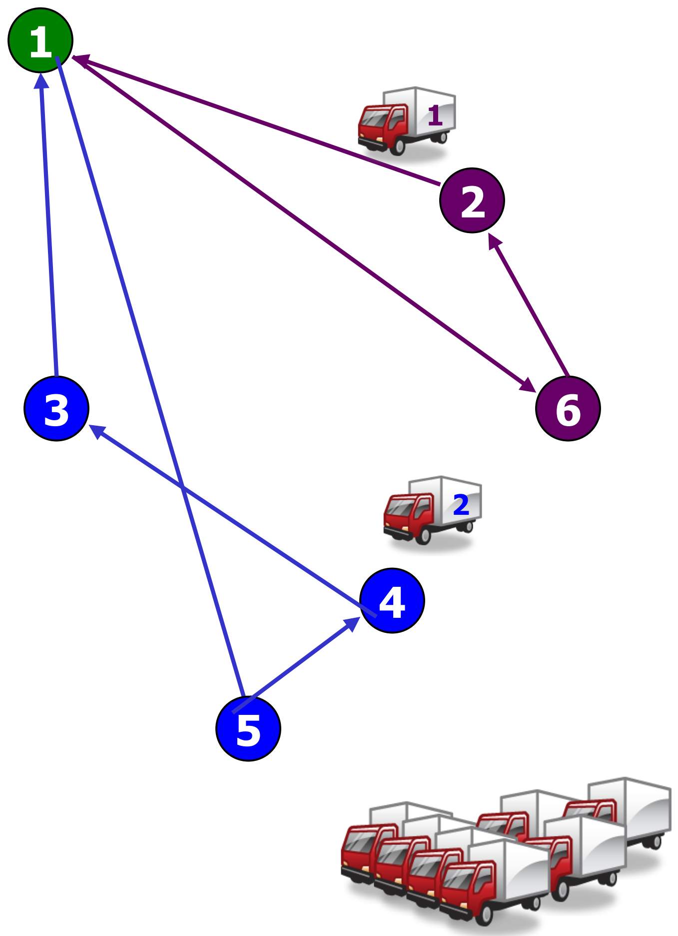


1. You must select the set of nodes that make up the route/path

2. You must select the sequence of nodes within the route

(TSP)

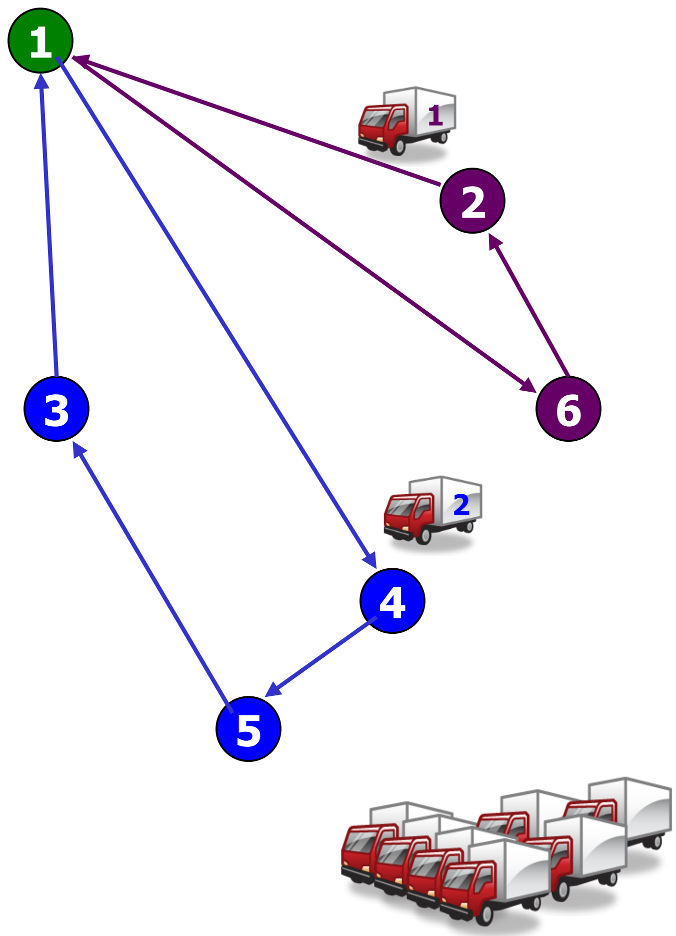
VRP: VEHICLE ROUTING PROBLEM



1. You must select the set of nodes that make up the route/path

2. You must select the sequence of nodes within the route (TSP)

VRP: VEHICLE ROUTING PROBLEM



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv}\ +\Sigma_v\, d_v y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq\ y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

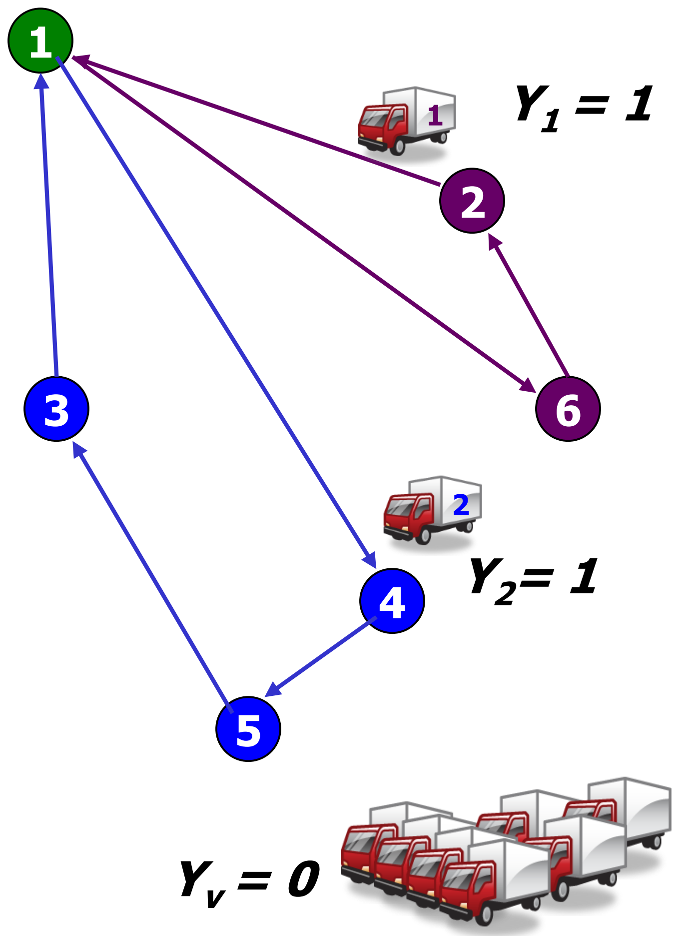
d_v Cost activate route v

c_{ijv} Cost of going from i to j using the route v

y_v Decision to activate the route v

x_{ijrv} Decision to go from i to j using route v

VRP: VEHICLE ROUTING PROBLEM



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

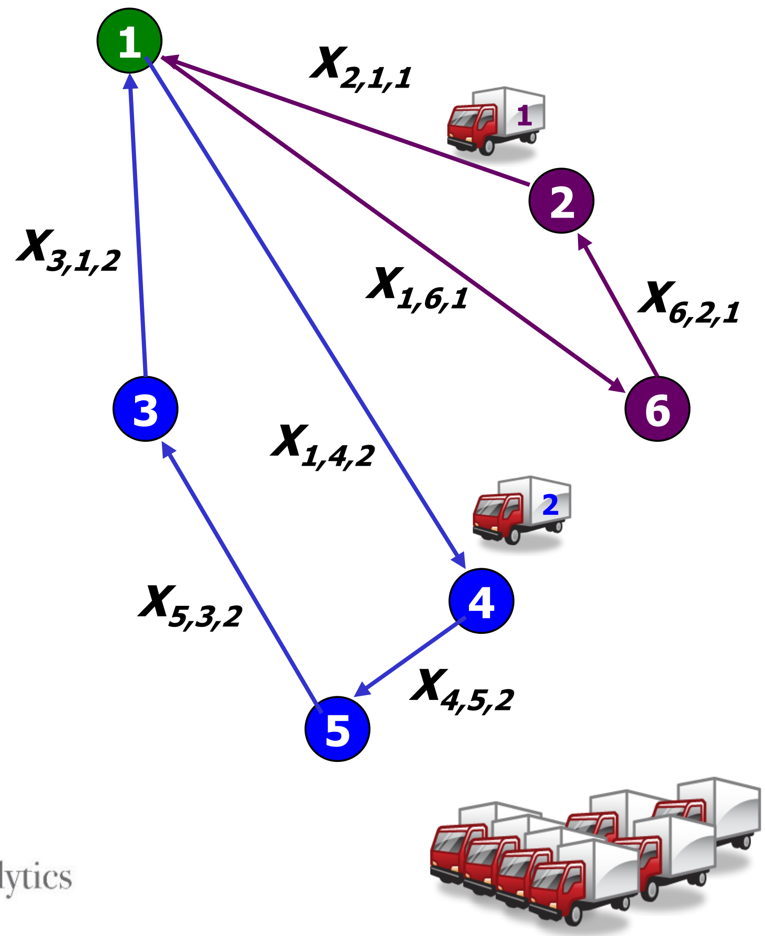
d_v Cost activate route v

c_{ijv} Cost of going from i to j using the route v

y_v **Decision to activate the route v**

x_{ijrv} Decision to go from i to j using route v

VRP: VEHICLE ROUTING PROBLEM



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

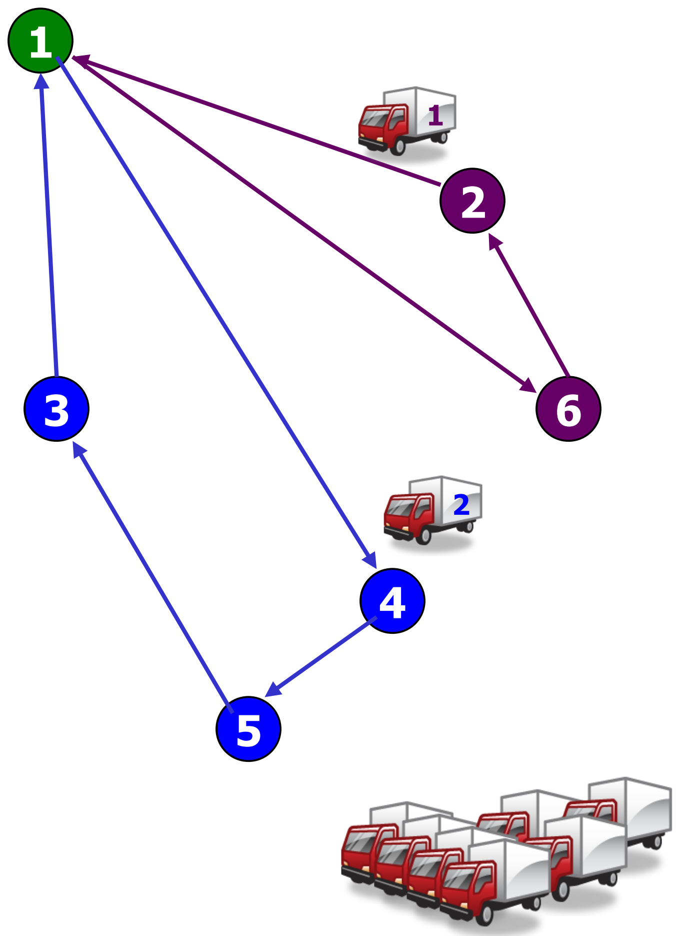
d_v Cost activate route v

c_{ijv} Cost of going from i to j using the route v

y_v **Decision to activate the route v**

x_{ijrv} Decision to go from i to j using route v

VRP: VEHICLE ROUTING PROBLEM



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

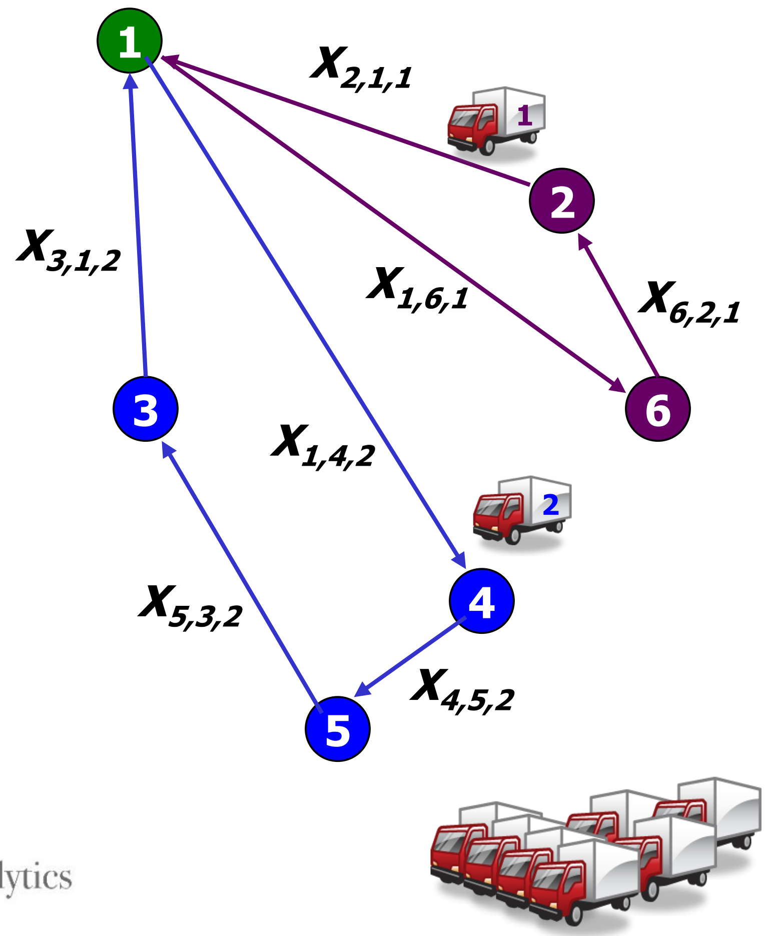
d_v Cost activate route v

c_{ijv} Cost of going from i to j using the route v

y_v Decision to activate the route v

x_{ijrv} Decision to go from i to j using route v

VRP: VEHICLE ROUTING PROBLEM



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v\, y_v$$

Subjet to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \ \ \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

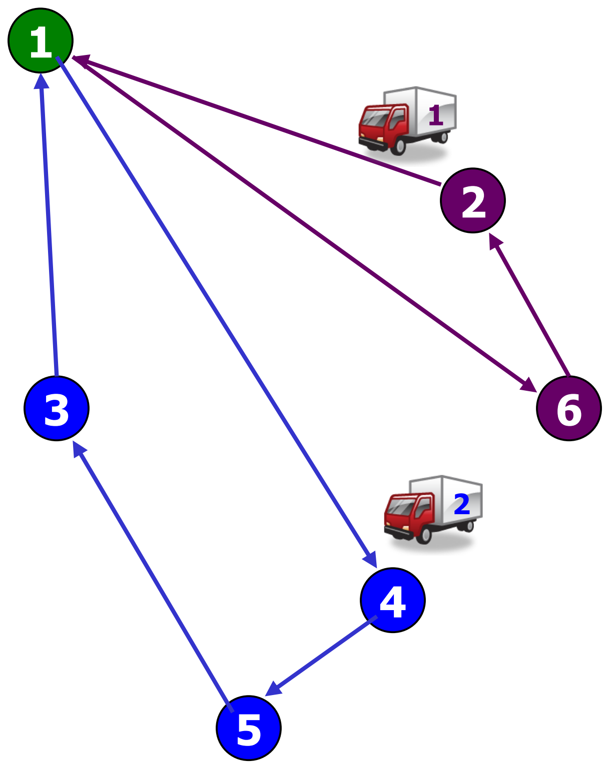
d_v Cost activate route v

c_{ijv} Cost of going from i to j using the route v

y_v **Decision to activate the route v**

x_{ijrv} **Decision to go from i to j using route v**

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

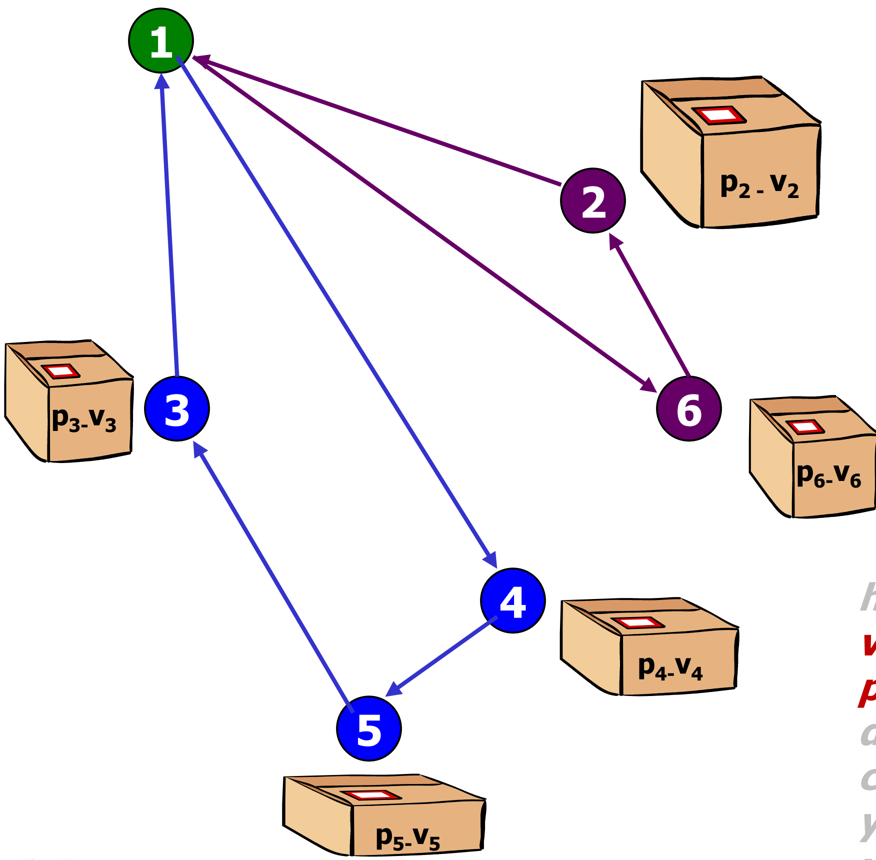
$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq Weight_v \quad \forall v$$

h_{ijv} Travel time from i to j on route v (hr)
v_i Weight associated with the order in i (kg)
p_i Volume associated with the order in i (m3)
d_v Cost activate route v
c_{ijv} Cost of going from i to j using the route v
y_v Decision to activate the route v
x_{ijrv} Decision to go from i to j using route v

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS



$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq Weight_v \quad \forall v$$

h_{ijv}	*Travel time from i to j on route v (hr)*
v_i	***Weight associated with the order in i (kg)***
p_i	***Volume associated with the order in i (m3)***
d_v	*Cost activate route v*
c_{ijv}	*Cost of going from i to j using the route v*
y_v	*Decision to activate the route v*
x_{ijrv}	*Decision to go from i to j using route v*

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq Weight_v \quad \forall v$$

MATHEMATICAL ELEMENTS/OBJECTS

INDEX

SET

PARAMETER

VARIABLE

CONSTRAINT

OBJECTIVE FUNCTION

PROBLEM

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv}\ + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \ \ \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq Weight_v \quad \forall v$$

MATHEMATICAL ELEMENTS/OBJECTS

INDEXES:

i	*Node/Client*
j	*Node/Client*
v	*Route/Path/Vehicle*

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv}\ + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \ \ \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq\ y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq\ Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq\ Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq\ Weight_v \quad \forall v$$

MATHEMATICAL ELEMENTS/OBJECTS

SETS:

IMPLICIT:

i ***All Nodes/Clients***
j ***All Nodes /Clients***
v ***All Routes/Paths/Vehicles***

EXPLICIT:

$\forall$ ***i*** ***All Nodes/Clients***
$\forall$ ***j*** ***All Nodes /Clients***
$\forall$ ***v*** ***All Routes/Paths/Vehicles***

$\forall i \neq 1$ ***All nodes except the "default node" warehouse***

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$Min\ \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv}\ + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq Weight_v \quad \forall v$$

MATHEMATICAL ELEMENTS/OBJECTS

PARAMETERS:

- h_{ijv} — Travel time from i to j on route v (hr)
- v_i — Weight associated with the order in i (kg)
- p_i — Volume associated with the order in i (m3)
- d_v — Cost activate route v ($)
- c_{ijv} — Cost of going from i to j using the route v ($)

- $Time_v$ — Time available for route v
- Volume — Volume capacity of route v
- $Weight_v$ — Weight capacity of route v

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$\text{Min } \Sigma_i \Sigma_j \; c_{ij} \, x_{ijv} \; + \Sigma_v \, d_v \, y_v$$

Subject to

$$\Sigma_j \Sigma_v \, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j \, x_{jiv} = \Sigma_j \, x_{jiv} \quad \forall i \;\; \forall v$$

$$\Sigma_i \Sigma_j \, x_{ijv} \leq \; y_v \quad \forall v$$

$$y_v \in \{0,1\}, \; x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j \;\; h_{ijv} \, x_{ijr} \leq \; Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j \;\; v_i \, x_{ijr} \leq \; Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j \;\; p_i \, x_{ijr} \leq \; Weight_v \quad \forall v$$

MATHEMATICAL ELEMENTS/OBJECTS

VARIABLES:

y_v	Decision to activate the route v (binary)
x_{ijrv}	Decision to go from i to j using route v (binary)

The variables are restricted by its type and its existence conditions

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$\text{Min } \Sigma_i \Sigma_j \; c_{ij} x_{ijv} + \Sigma_v d_v y_v$$

Subject to

$CO1_i$	$\Sigma_j \Sigma_v x_{ijv} = 1$	$\forall i \neq 1$
$CO2_{iv}$	$\Sigma_j x_{jiv} = \Sigma_j x_{jiv}$	$\forall i \;\; \forall v$
$CO3_v$	$\Sigma_i \Sigma_j x_{ijv} \leq y_v$	$\forall v$
	$y_v \in \{0,1\}, \; x_{ijv} \in \{0,1\}$	
	$x_{iiv} = 0$	
TIM_v	$\Sigma_i \Sigma_j \; h_{ijv} x_{ijr} \leq Time_v$	$\forall v$
VOL_v	$\Sigma_i \Sigma_j \; v_i x_{ijr} \leq Volume_v$	$\forall v$
WEI_v	$\Sigma_i \Sigma_j \; p_i x_{ijr} \leq Weight_v$	$\forall v$

MATHEMATICAL ELEMENTS/OBJECTS

CONSTRAINTS:

$CO1_i$	All nodes must visit once
$CO2_{iv}$	If one route arrive to the node i must leave from this node.
$CO3_v$	Only if the route is activated can visit a node
TIM_v	The sum of the travel times must be less than permitted time for the route (hr)
VOL_v	The sum of the volumes transported must be less than the volume capacity of the route (m3)
WEI_v	The sum of the weights transported must be less than the weight capacity of the route (Kg)

The constraint must satisfies its existence conditions

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

$$Min\ \ cfv = \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv} + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq Weight_v \quad \forall v$$

MATHEMATICAL ELEMENTS/OBJECTS

OBJECTIVE FUNCTION:

cfv ***Sum of fixed and variables cost ($)***

VRP: VEHICLE ROUTING PROBLEM WITH RESOURCES CONSTRAINTS
SYSTEM INFORMATION APPROACH

VRPTVW:= {

$$Min\ \ cfv = \Sigma_i \Sigma_j\ c_{ij}\, x_{ijv}\ + \Sigma_v\, d_v\, y_v$$

Subject to

$$\Sigma_j \Sigma_v\, x_{ijv} = 1 \quad \forall i \neq 1$$

$$\Sigma_j\, x_{jiv} = \Sigma_j\, x_{jiv} \quad \forall i \quad \forall v$$

$$\Sigma_i \Sigma_j\, x_{ijv} \leq\ y_v \quad \forall v$$

$$y_v \in \{0,1\},\ x_{ijv} \in \{0,1\}$$

$$x_{iiv} = 0$$

$$\Sigma_i \Sigma_j\ h_{ijv}\, x_{ijr} \leq\ Time_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ v_i\, x_{ijr} \leq\ Volume_v \quad \forall v$$

$$\Sigma_i \Sigma_j\ p_i\, x_{ijr} \leq\ Weight_v \quad \forall v$$

}

MATHEMATICAL ELEMENTS/OBJECTS

PROBLEM:

VRPTVW ***Vehicle Routing problem with constraint of time, volume and weight.***

OPTEX Mathematical Modeling System - Chief Scientist DecisionWare International Corp. (OPTEX MMS 374838-456059)

TUTORIAL

DO Analytics

Vehicle Routing Problem

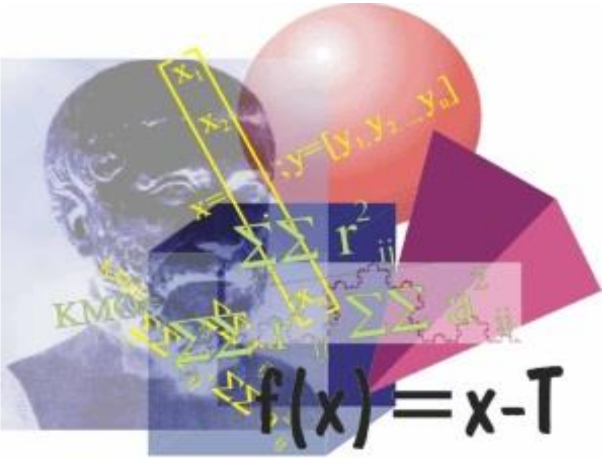
Powered by

Think the mathematical model and OPTΣX will make the software for you

Mathematical
Modeling System

Aceptar | Usuario VRP | Clave | Cambiar Clave | Nueva Clave | Cancelar

DATABASE ALGEBRAIC LANGUAGE
(THE NEW PARADIGM)



CLICK OVER THE IMAGE TO OBTAIN MORE INFORMATION

OPTEX sees the implementation of a Decision Support System as a load of a Relational Information System converting the mathematical modeling and the software production in a “filling the blanks” process.

DOA suggests that you review the presentation related to the algebraic language based on tables, exclusive of OPTEX. To do this you can download the presentation of the web

PDF DOWNLOAD

Analytics

The terminology used in mathematical distribution system modeling is defined:

- **Vehicle:** Means of transport to be used to provide the services which has a capacity expressed in terms of weight (kg) and volume (m3); and costs, fixed and variables, associated with the use of the vehicle.

- **Node:** Spot in the geo-referenced map that represents the warehouse, and the clients, these must be visited by a vehicle to provide a service of loading/unloading of goods which have associated the following characteristics:
  - Amount of goods (demand) of different types of goods to be delivered or collected
  - Hours of receipt or delivery of goods.
  - Time averages required for the loading or unloading of the goods.
  - Restrictions on mobility of vehicles to customers by urban standards or by rules on the client's sites

- **Road Arc:** Arch of the road grid that can be used by a vehicle when it is moving between two nodes.

- **Path:** Set of road segments between two nodes.

- **Route:** Sequence of nodes/clients that the vehicle should visit to provide services.

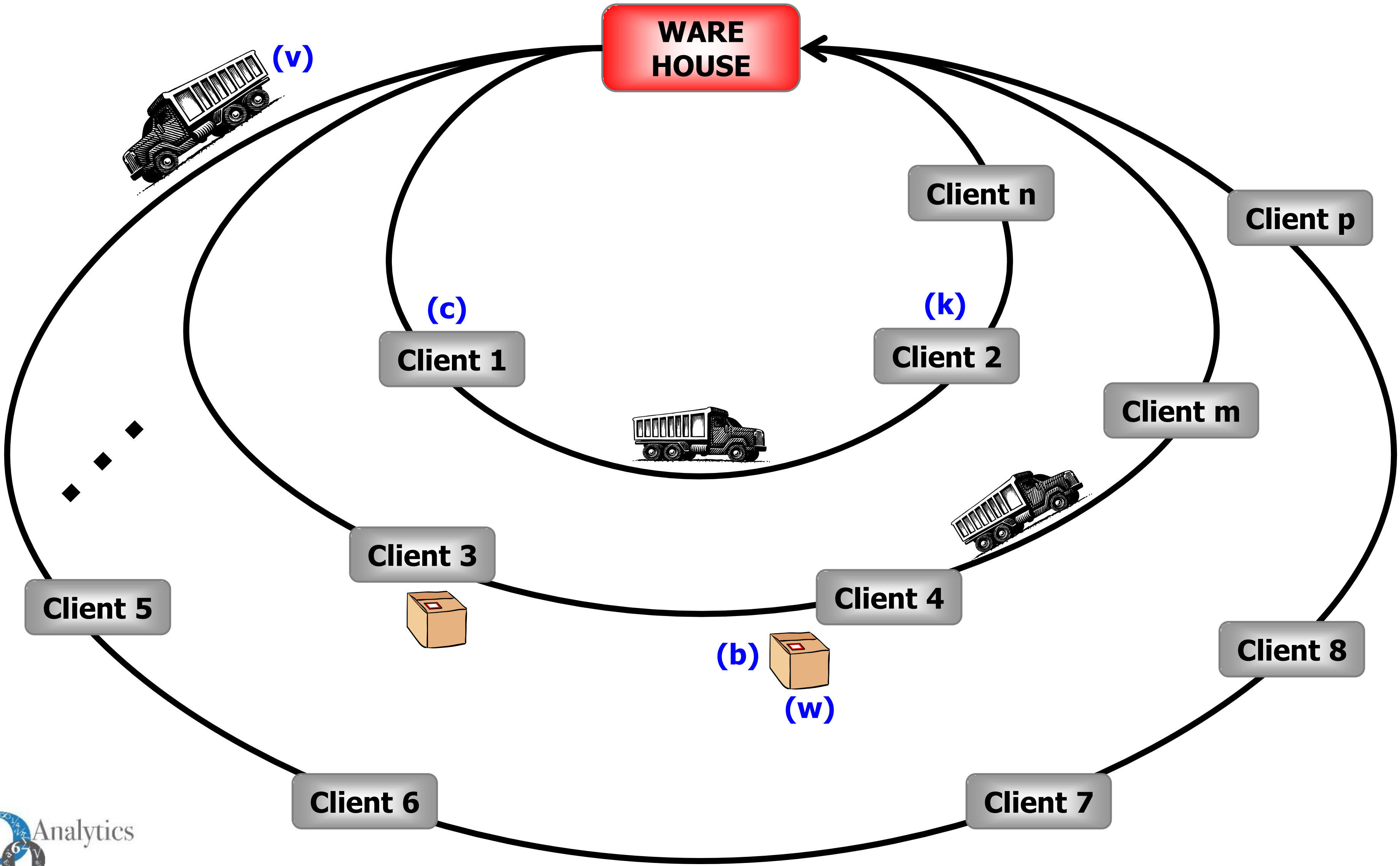
- **Zone:** Grouping of nodes whose demand for services should be attended by a set of vehicles assigned to the zone

Mathematical modelling represents the flow of merchandise between the nodes of the distribution system: warehouse (logistic operator) and clients (c).

The orders (w) of the clients (c) activate the distribution system, these determine the routes and the allocation of system resources. The logistics operator handles a variety of goods from different customers that are stored in the warehouse (c) contained in boxes (b) that have associated a weight and volume. For the distribution and transport of goods, the logistics operator has a heterogeneous fleet of vehicles (v), own and third parties, that have associated a load capacity in weight and volume.

Vehicles (v) are involved in a main activity that is the distribution of goods, this activity is to deliver the orders from the warehouse of the logistics operator (c) to the clients (c). The fleet of vehicles (v) has transit restrictions, what prevents that certain vehicles (v) can go to certain clients (c).

DISTRIBUTION SYSTEM



We considered two problems:

1. **A simple VRP model that determines the optimal route of vehicles (sequence of customers to visit) in an area without considering additional restrictions;**

2. **Posteriori, we include two constraints of capacity, one volume and one of weight are included, this new model is called VRP2C.**

FORMULATION AND IMPLEMENTATION OF THE VRP MODEL

Since implementation of the mathematical model in OPTEX implies its storage in a relational information system, whereas the reader to approach the organization of the formulation of mathematical models from this point of view.

Therefore, if an information system is a set of "related" tables it is convenient that the first version of the mathematical model approach is based on tables that contain the different elements that compose the algebraic formulation.

Then the VRP model is presented in the following elements: indexes, sets, parameters, variables, constraints and objective functions; from the previous concepts is parameterized higher level elements, such as: problems and models.

INDEXES

The entities that are handled in the model must be associated with indexes that represent them in the algebraic formulation.

Each type of entity requires at least an index to represent it, when there are mathematical elements that relate two physical entities of the same type is required to define indexes "alias" for the correct formulation of the models.

INDEXES						
INDEX	**ENTITY OBJECT**	**DESCRIPTION**	**ALIAS**	**MASTER TABLE**	**SCENARIO TABLE**	**RELATIONAL KEY**
b	**Boxes**	Container in which it is protected, stored and transported merchandise		CAJAS	ESC_CAJ	COD_CAJ
c	**Node**	Spatial point that must be visited by a vehicle to provide a service of loading and/or unloading of goods	**k**	NODOS	ESC_NOD	COD_NOD
k	**Node (Alias)**	Spatial point that must be visited by a vehicle to provide a service of loading and/or unloading of goods	**c**	NODOS	ESC_NOD1	COD_NOD1
v	**Vehicle**	Transport equipment to be used to provide transportation services		VEHICULOS	ESC_VEH	COD_VEH
w	**Orders**	Custom merchandise that customers make and must be shipped and transported		PEDIDOS	ESC_PED	COD_PED

INDEXES

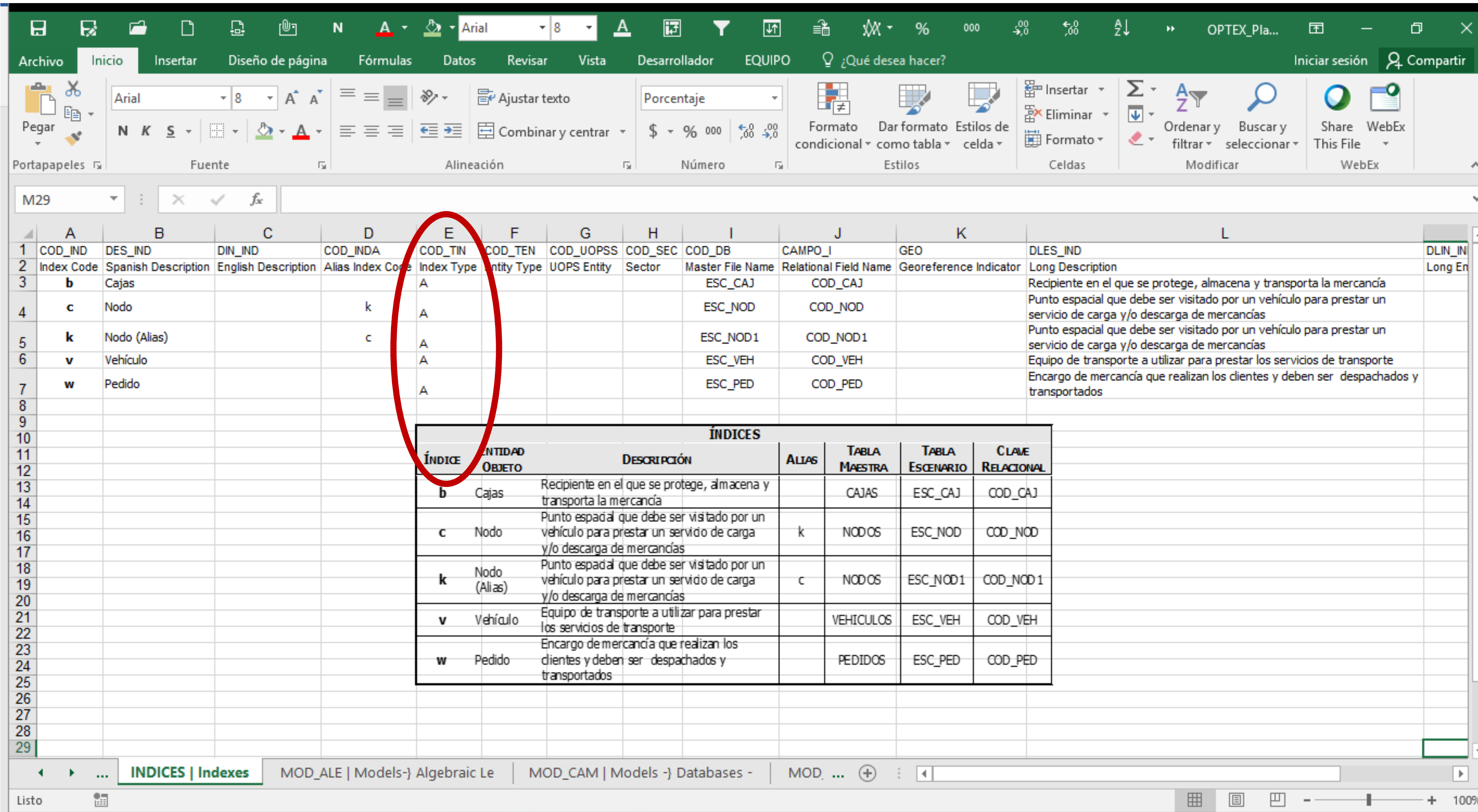
In addition, each index should link a master table that stores the attributes of physical entities associated with the index; to store the codes assigned to the physical entities, you must define a relational key, which will be the element that establishes the relationships between different tables that make up the information system of the data of the problem.

Given that a master table can contain physical entities that are not considered in the model, it is necessary to define a reference table containing physical entities to be included in the model, these tables are called as scenario table and define the topology of the mathematical model to solve, and therefore are associated with a case or scenario.

INDEXES						
Index	**Entity Object**	**Description**	**Alias**	**Master Table**	**Scenario Table**	**Relational Key**
b	Boxes	Container in which it is protected, stored and transported merchandise		**CAJAS**	**ESC_CAJ**	COD_CAJ
c	Node	Spatial point that must be visited by a vehicle to provide a service of loading and/or unloading of goods	k	**NODOS**	**ESC_NOD**	COD_NOD
k	Node (Alias)	Spatial point that must be visited by a vehicle to provide a service of loading and/or unloading of goods	c	**NODOS**	**ESC_NOD1**	COD_NOD1
v	Vehicle	Transport equipment to be used to provide transportation services		**VEHICULOS**	**ESC_VEH**	COD_VEH
w	Orders	Custom merchandise that customers make and must be shipped and transported		**PEDIDOS**	**ESC_PED**	COD_PED

INDEXES

The following image presents the index information loaded in the EXCEL template (sheet INDICES). In this case we have included additional information corresponding to the type of index (COD_TIN), which is A (alphanumeric) for all cases.



COD_IND	DES_IND	DIN_IND	COD_INDA	COD_TIN	COD_TEN	COD_UOPSS	COD_SEC	COD_DB	CAMPO_I	GEO	DLES_IND	DLIN_IN
Index Code	Spanish Description	English Description	Alias Index Code	Index Type	Entity Type	UOPS Entity	Sector	Master File Name	Relational Field Name	Georeference Indicator	Long Description	Long En
b	Cajas			A				ESC_CAJ	COD_CAJ		Recipiente en el que se protege, almacena y transporta la mercancía	
c	Nodo		k	A				ESC_NOD	COD_NOD		Punto espacial que debe ser visitado por un vehículo para prestar un servicio de carga y/o descarga de mercancías	
k	Nodo (Alias)		c	A				ESC_NOD1	COD_NOD1		Punto espacial que debe ser visitado por un vehículo para prestar un servicio de carga y/o descarga de mercancías	
v	Vehículo			A				ESC_VEH	COD_VEH		Equipo de transporte a utilizar para prestar los servicios de transporte	
w	Pedido			A				ESC_PED	COD_PED		Encargo de mercancía que realizan los clientes y deben ser despachados y transportados	

ÍNDICES

Índice	Entidad Objeto	Descripción	Alias	Tabla Maestra	Tabla Escenario	Clave Relacional
b	Cajas	Recipiente en el que se protege, almacena y transporta la mercancía		CAJAS	ESC_CAJ	COD_CAJ
c	Nodo	Punto espacial que debe ser visitado por un vehículo para prestar un servicio de carga y/o descarga de mercancías	k	NODOS	ESC_NOD	COD_NOD
k	Nodo (Alias)	Punto espacial que debe ser visitado por un vehículo para prestar un servicio de carga y/o descarga de mercancías	c	NODOS	ESC_NOD1	COD_NOD1
v	Vehículo	Equipo de transporte a utilizar para prestar los servicios de transporte		VEHICULOS	ESC_VEH	COD_VEH
w	Pedido	Encargo de mercancía que realizan los clientes y deben ser despachados y transportados		PEDIDOS	ESC_PED	COD_PED

VARIABLES

The variables of the problem are associated with the decisions the final user of the mathematical models. In this case the modeled decisions are:

- **Activation of the service for a vehicle.**
- **Order of visit of vehicles to the destination.**

The conditions of existence of a variable determines the values of combinations of indexes (physical entities) for which the variable exists and is determined based on the sets that define them.

Below, is the set of required binary variables:

VARIABLES				
VARIABLE	**DESCRIPTION**	**MEASURE UNIT**	**TYPE**	**EXISTENCE CONDITIONS**
$\mathbf{AVL_v}$	**Using vehicle v** Binary variable that determines whether to use the vehicle v to meet the orders of customers. Exists for all vehicle v in the problem, which is represented by the set **v∈VEH**.		**B**	**∀v∈VEH**
$\mathbf{VCL_{v,c,k}}$	**Vehicle v goes from c to k** Binary variable that determines if the vehicle goes from the node origin **c** to node destination **k**. Exists for all vehicle **v** in the problem, all client **c** that can be attended by the vehicle **v** (**c∈NCV(v)**) and by all node **k** which can be visited from the node **c** in vehicle **v** (**k∈TRK(c,v)**)		**B**	**∀v∈VEH ∀c∈NCV(v) ∀k∈TRK(c,v)**

VARIABLES

The definition of the variables in OPTEX implies filling two tables:

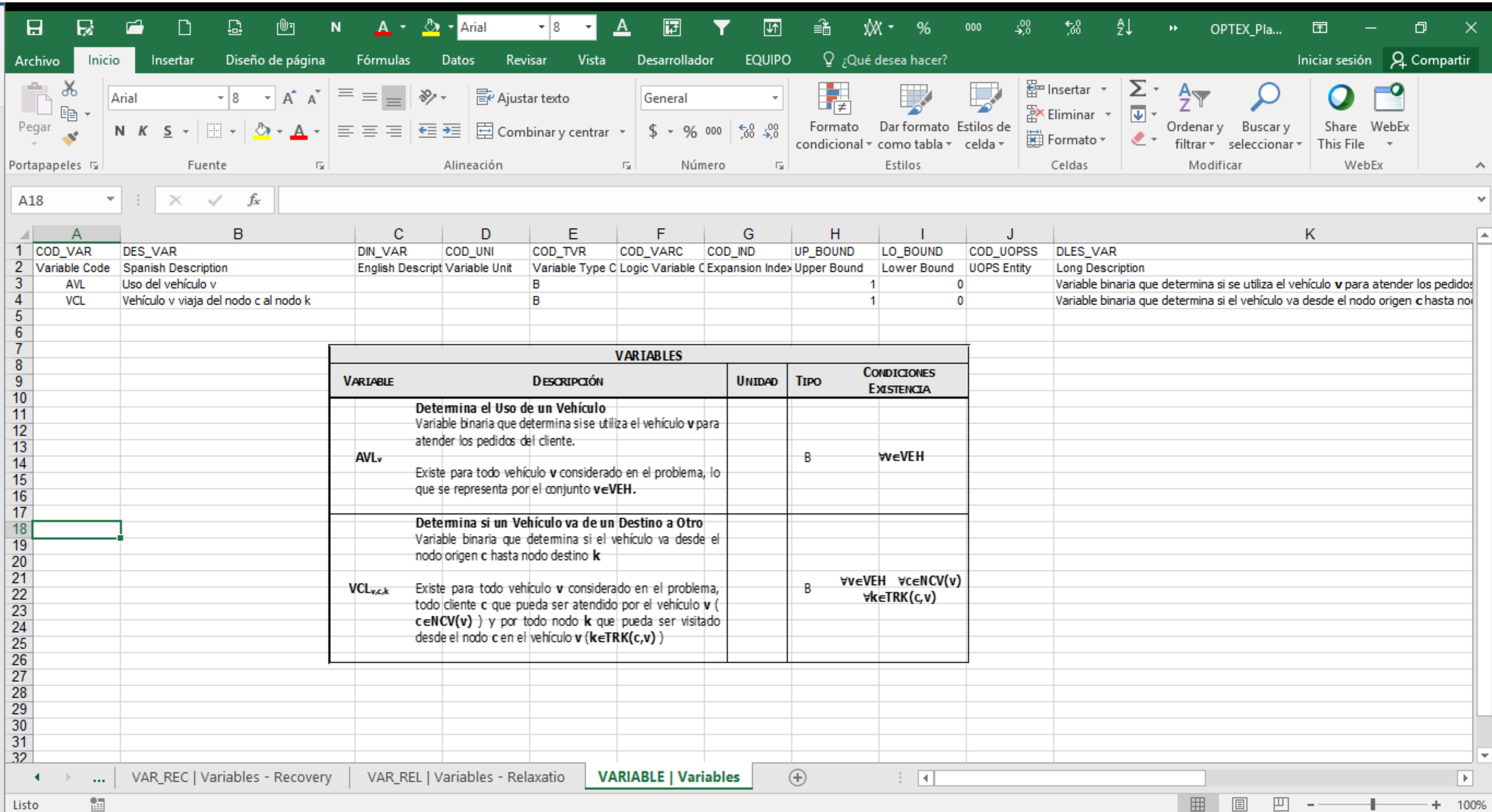
1. **The first (sheet VARIABLE) which determines the general attributes of the variable**

2. **The second (sheet VAR_IND) to define the indexes of variables and their conditions of existence.**

VARIABLES				
VARIABLE	**DESCRIPTION**	**MEASURE UNIT**	**TYPE**	**EXISTENCE CONDITIONS**
AVL_v	**Using vehicle v** Binary variable that determines whether to use the vehicle v to meet the orders of customers. Exists for all vehicle v in the problem, which is represented by the set **v∈VEH**.		B	∀v∈VEH
$VCL_{v,c,k}$	**Vehicle v goes from c to k** Binary variable that determines if the vehicle goes from the node origin **c** to node destination **k**. Exists for all vehicle **v** in the problem, all client **c** that can be attended by the vehicle **v** (**c∈NCV(v)**) and by all node **k** which can be visited from the node **c** in vehicle **v** (**k∈TRK(c,v)**)		B	∀v∈VEH ∀c∈NCV(v) ∀k∈TRK(c,v)

VARIABLES

1. **The first (sheet VARIABLE) which determines the general attributes of the variable**

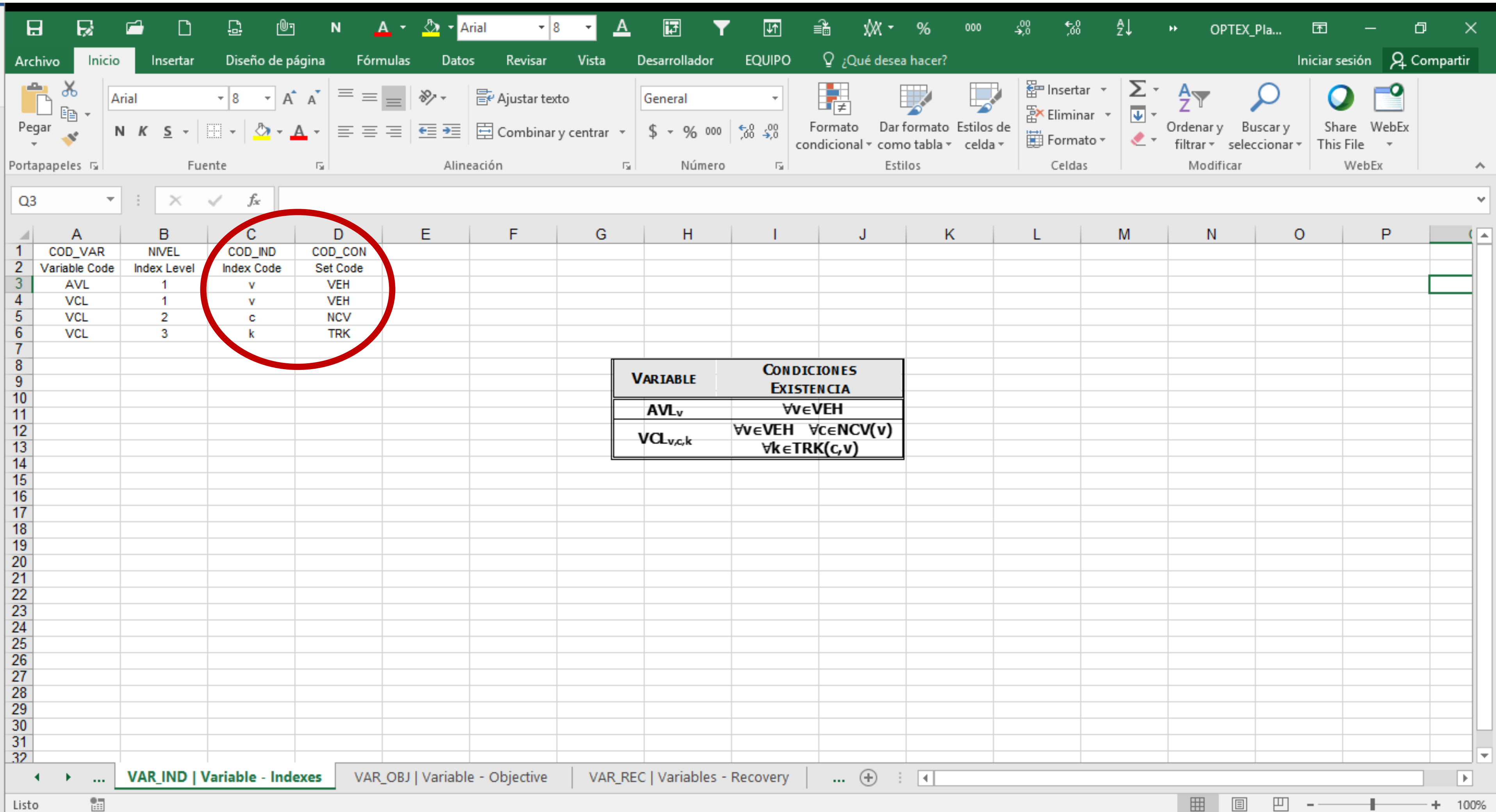


COD_VAR	DES_VAR	DIN_VAR	COD_UNI	COD_TVR	COD_VARC	COD_IND	UP_BOUND	LO_BOUND	COD_UOPSS	DLES_VAR
Variable Code	Spanish Description	English Descript	Variable Unit	Variable Type C	Logic Variable C	Expansion Index	Upper Bound	Lower Bound	UOPS Entity	Long Description
AVL	Uso del vehículo v			B			1	0		Variable binaria que determina si se utiliza el vehículo **v** para atender los pedidos
VCL	Vehículo v viaja del nodo c al nodo k			B			1	0		Variable binaria que determina si el vehículo va desde el nodo origen **c** hasta no

VARIABLES				
Variable	**Descripción**	**Unidad**	**Tipo**	**Condiciones Existencia**
AVL_v	**Determina el Uso de un Vehículo** Variable binaria que determina si se utiliza el vehículo **v** para atender los pedidos del cliente. Existe para todo vehículo **v** considerado en el problema, lo que se representa por el conjunto **v∈VEH.**		B	**∀v∈VEH**
$VCL_{v,c,k}$	**Determina si un Vehículo va de un Destino a Otro** Variable binaria que determina si el vehículo va desde el nodo origen **c** hasta nodo destino **k** Existe para todo vehículo **v** considerado en el problema, todo cliente **c** que pueda ser atendido por el vehículo **v** (**c∈NCV(v)**) y por todo nodo **k** que pueda ser visitado desde el nodo **c** en el vehículo **v** (**k∈TRK(c,v)**)		B	**∀v∈VEH ∀c∈NCV(v) ∀k∈TRK(c,v)**

VARIABLES

2. The second (sheet **VAR_IND**) to define the indexes of variables and their **conditions of existence**.



A	B	C	D
COD_VAR	NIVEL	COD_IND	COD_CON
Variable Code	Index Level	Index Code	Set Code
AVL	1	v	VEH
VCL	1	v	VEH
VCL	2	c	NCV
VCL	3	k	TRK

VARIABLE	CONDICIONES EXISTENCIA
AVL_v	$\forall v \in VEH$
$VCL_{v,c,k}$	$\forall v \in VEH \quad \forall c \in NCV(v)$ $\forall k \in TRK(c,v)$

CONSTRAINTS

The constraints required to model the problem VRP are presented (the detailed formulation is in the Tutorial Manual):

CONSTRAINTS- VRP MODEL	
CONSTRAINT	**DESCRIPTION- EQUATION**
$\mathbf{SANO_{v,c}}$	**Departure from origin node** $\Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - AVL_v = 0$ $\forall v \in VEH \quad \forall c \in NOV(v)$
$\mathbf{ENSA_{v,c}}$	**Arrival and departures of a node** $\Sigma_{k \in TRK(c,v)} VCL_{v,k,c} - \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} = 0$ $\forall v \in VEH \quad \forall c \in NCV(v)$
$\mathbf{UTVE_{v}}$	**Use of vehicles** $\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - \infty \times AVL_v \leq 0$ $\forall v \in VEH$
$\mathbf{VCLI_{c}}$	**Visit of destination** $\Sigma_{v \in VEC(c)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} = 1$ $\forall c \in DEC$
$\mathbf{CAPP_{v}}$	**Load capacity of vehicles** $\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c\ v)} DEMP_c \times VCL_{v,c,k} \leq CAPP_v$ $\forall v \in VEH$
$\mathbf{CAPV_{v}}$	**Volumetric capacity of vehicles** $\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c\ v)} DEMV_c \times VCL_{v,c,k} \leq CAPV_v$ $\forall v \in VEH$

CONSTRAINTS

The definition of the constraints in OPTEX implies filling three tables:

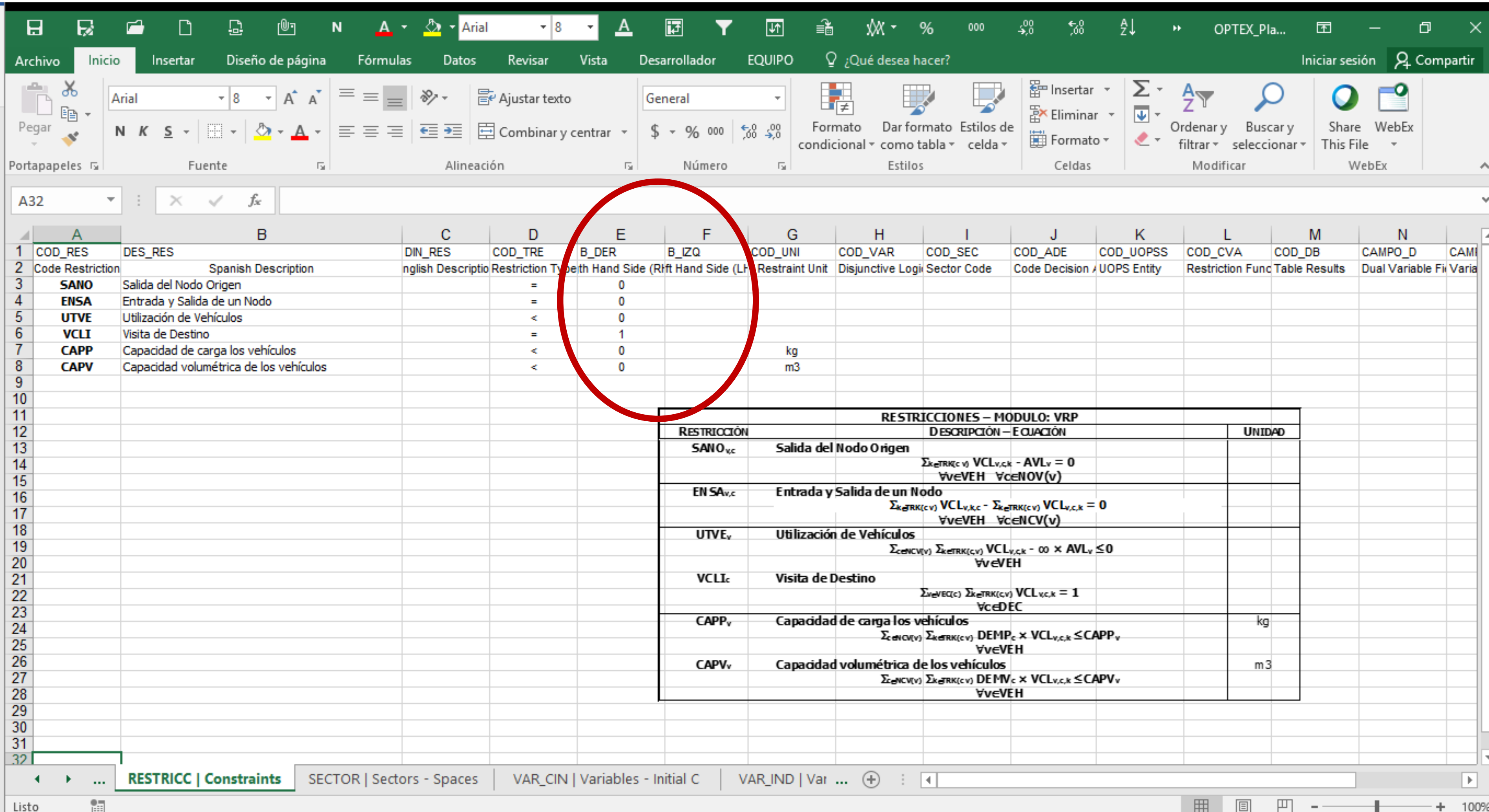
1. **The first (sheet RESTRICC) which determines the general attributes of the constraints**

2. **The second (sheet RES_IND) to define the indexes of the constraint and their conditions of existence.**

3. **The third (sheet ECUACION) which contains the terms that break down every equation of the model.**

CONSTRAINTS

1. The first (sheet **RESTRICC**) which determines the general attributes of the constraints; it includes the definition of the **RHS (right hand side)** and **LHS (left hand side)** of the constraint.



A	B	C	D	E	F	G	H	I	J	K	L	M	N
COD_RES	DES_RES	DIN_RES	COD_TRE	B_DER	B_IZQ	COD_UNI	COD_VAR	COD_SEC	COD_ADE	COD_UOPSS	COD_CVA	COD_DB	CAMPO_D
Code Restriction	Spanish Description	nglish Descriptio	Restriction Ty	th Hand Side (R	ft Hand Side (LH	Restraint Unit	Disjunctive Logi	Sector Code	Code Decision A	UOPS Entity	Restriction Func	Table Results	Dual Variable Fi
SANO	Salida del Nodo Origen		=	0									
ENSA	Entrada y Salida de un Nodo		=	0									
UTVE	Utilización de Vehículos		<	0									
VCLI	Visita de Destino		=	1									
CAPP	Capacidad de carga los vehículos		<	0		kg							
CAPV	Capacidad volumétrica de los vehículos		<	0		m3							

RESTRICCIONES – MODULO: VRP

RESTRICCIÓN	DESCRIPCIÓN – ECUACIÓN	UNIDAD
$SANO_{v,c}$	Salida del Nodo Origen $\Sigma_{k\in TRK(c,v)} VCL_{v,c,k} - AVL_v = 0$ $\forall v\in VEH \quad \forall c\in NOV(v)$	
$ENSA_{v,c}$	Entrada y Salida de un Nodo $\Sigma_{k\in TRK(c,v)} VCL_{v,k,c} - \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 0$ $\forall v\in VEH \quad \forall c\in NCV(v)$	
$UTVE_v$	Utilización de Vehículos $\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} - \infty \times AVL_v \leq 0$ $\forall v\in VEH$	
$VCLI_c$	Visita de Destino $\Sigma_{v\in VEC(c)} \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 1$ $\forall c\in DEC$	
$CAPP_v$	Capacidad de carga los vehículos $\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c,v)} DEMP_c \times VCL_{v,c,k} \leq CAPP_v$ $\forall v\in VEH$	kg
$CAPV_v$	Capacidad volumétrica de los vehículos $\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c,v)} DEMV_c \times VCL_{v,c,k} \leq CAPV_v$ $\forall v\in VEH$	m3

RESTRICC | Constraints   SECTOR | Sectors - Spaces   VAR_CIN | Variables - Initial C   VAR_IND | Var

CONSTRAINTS

2. **The second (sheet RES_IND) to define the indexes of the constraint and their conditions of existence.**

COD_RES	NIVEL	COD_IND	COD_CON
Constraint Code	Index Level	Index Code	Set Code
SANO	1	v	VEH
SANO	2	c	NOV
ENSA	1	v	VEH
ENSA	2	c	NCV
UTVE	1	v	VEH
VCLI	1	c	DEC
CAPP	1	v	VEH
CAPP	1	v	VEH

RESTRICCIONES – MODULO: VRP		
RESTRICCIÓN	DESCRIPCIÓN – ECUACIÓN	UNIDAD
$SANO_{v,c}$	Salida del Nodo Origen $\Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - AVL_v = 0$ $\forall v \in VEH \quad \forall c \in NOV(v)$	
$ENSA_{v,c}$	Entrada y Salida de un Nodo $\Sigma_{k \in TRK(c,v)} VCL_{v,k,c} - \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} = 0$ $\forall v \in VEH \quad \forall c \in NCV(v)$	
$UTVE_v$	Utilización de Vehículos $\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - \infty \times AVL_v \leq 0$ $\forall v \in VEH$	
$VCLI_c$	Visita de Destino $\Sigma_{v \in VEC(c)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} = 1$ $\forall c \in DEC$	
$CAPP_v$	Capacidad de carga los vehículos $\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} DEMP_c \times VCL_{v,c,k} \leq CAPP_v$ $\forall v \in VEH$	kg
$CAPV_v$	Capacidad volumétrica de los vehículos $\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} DEMV_c \times VCL_{v,c,k} \leq CAPV_v$ $\forall v \in VEH$	m3

RES_IND | Constraint - Indexes   RES_REC | Constraints - Recover   RES_REL | Constraints - Relaxat

OPTEX
Mathematical Modeling System

CONSTRAINTS

3. **The third (sheet ECUACION) which contains the terms that break down every equation of the constraints of the model. Given that this process is the "hardest" of the template, then, the accomplished process is analyzed restriction by restriction.**

COD_RES	SEQ	SIGNO	CAMPO_1	CAMPO_2	CAMPO_3	COD_VAR	COD_PAR
Constraint Code	Sequence	Sign (+ or -)	Contains: SUM c	Contains: Subsc	Contains: Variat	Variable Code	Code Parameter
SANO	1	+	S	k/TRK			
SANO	2	+	1	VCL			
SANO	3	-	1	AVL			
ENSA	1	+	S	k/TRK			
ENSA	2	+	1	VCL			
ENSA	3	-	S	k/TRK			
ENSA	4	+	1	VKL			
UTVE	1	+	S	c/NCV			
UTVE	2	+	S	k/TRK			
UTVE	3	+	1	VCL			
UTVE	4	-	INFI	AVL			
VCLI	1	+	S	v/VEC			
VCLI	2	+	S	k/TRK			
VCLI	3	+	1	VCL			
CAPP	1	+	S	c/VEC			
CAPP	2	+	S	k/TRK			
CAPP	3	+	DEMP	VCL			
CAPV	1	+	S	c/VEC			
CAPV	2	+	S	k/TRK			
CAPV	3	+	DEMV	VCL			

ECUACION | Equations · ESCENARI | Family of Scenarios · FOB_FOB | Multi-Criteria Object · FUNO

DATABASE ALGEBRAIC LENGUAJE

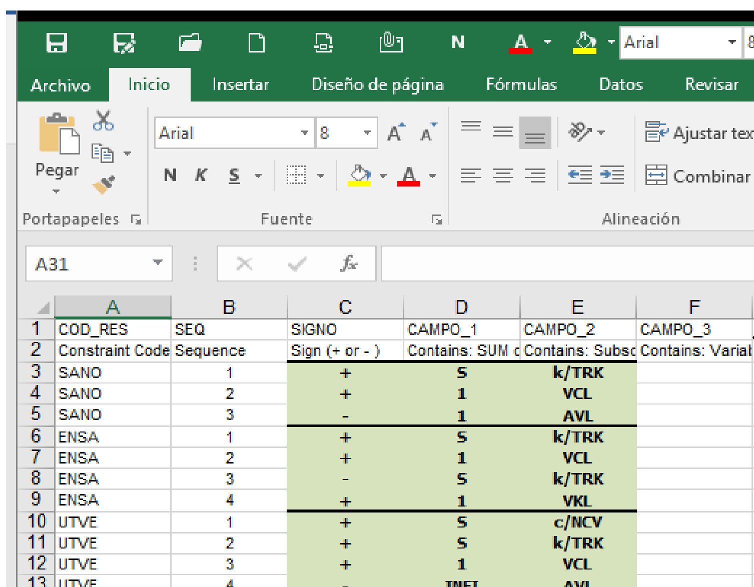
An algebraic language based on tables is used to describe the elements that integrate the equation.

For simplicity of presentation, it should be noted that this document describes only related to linear models as the VRP; However, it should be noted that the capabilities of the OPTEX database language allows the formulation of any non-linear expression.

Conceptually, an equation is considered as the sum of multiple terms each of which has five components:

- **SEQ**
- **SIGN (SIGNO)**
- **COMPONENT 1 (CAMPO_1)**
- **COMPONENT 2 (CAMPO_2)**
- **COMPONENT 3 (CAMPO_3)**

As its name suggests the SIGN determines if the evaluated expression will be multiplied by 1 or by -1 after being evaluated.



	A	B	C	D	E	F
1	COD_RES	SEQ	SIGNO	CAMPO_1	CAMPO_2	CAMPO_3
2	Constraint Code	Sequence	Sign (+ or -)	Contains: SUM c	Contains: Subsc	Contains: Variat
3	SANO	1	+	S	k/TRK	
4	SANO	2	+	1	VCL	
5	SANO	3	-	1	AVL	
6	ENSA	1	+	S	k/TRK	
7	ENSA	2	+	1	VCL	
8	ENSA	3	-	S	k/TRK	
9	ENSA	4	+	1	VKL	
10	UTVE	1	+	S	c/NCV	
11	UTVE	2	+	S	k/TRK	
12	UTVE	3	+	1	VCL	
13	UTVE	4	-	INFI	AVL	

DATABASE ALGEBRAIC LENGUAJE

An algebraic language based on tables is used to describe the elements that integrate the equation.

For simplicity of presentation, it should be noted that this document describes only related to linear models as the VRP; However, it should be noted that the capabilities of the OPTEX database language allows the formulation of any non-linear expression.

Conceptually, an equation is considered as the sum of multiple terms each of which has five components:

- **SEQ**
- **SIGN (SIGNO)**
- **COMPONENT 1 (CAMPO_1)**
- **COMPONENT 2 (CAMPO_2)**
- **COMPONENT 3 (CAMPO_3)**

Below, each component is described.

- **SEQ: determines the sequence of the terms of the equation.**

- **SIGN: determines if the evaluated expression will be multiplied by 1 or -1 after being evaluated.**

DATABASE ALGEBRAIC LENGUAJE

- **COMPONENT 1**
  The COMPONENT 1 can be:
  - **SUM, or S or Σ:** indicates that a sum is open. The elements on which the sum will be shown in COMPONENT 2.
  - **Numerical value:** for a constant numeric value (example: 1 or 43.56)
  - **Parameter:** corresponding to the value of a parameter, or a function of a parameter, which must be multiplied by the COMPONENT 2. The parameter subscripts are assumed to be equal to those defined for the parameter.

- **COMPONENT 2**
  The COMPONENT 2 can be:
  - **Limits of the sum:** corresponds to an expression that contains the information about the elements that should be included in the sum. The first element corresponds to the index that the sum will be held. The second corresponds to the set of reference to select the values of the index, the element should separate from the index using a slash (/). Alphanumeric index set is defined based on the code of the set. For numeric indexes (for example, the index t), the set is defined by the limits to which will vary the index, separated by a comma, in this case the / is replaced by an equal sign (=).
  - **Parameter:** corresponding to the name/code/ID of a parameter, or a function of a parameter, which must be multiplied by the COMPONENT 1. It is only applicable for formulas related parameters. The parameter subscripts are assumed to be equal to those defined for the parameter.
  - **Variable:** corresponding to the name/code/ID of a variable that has to be multiplied by the COMPONENT 1. The subscripts of the variable are assumed to be equal to those defined for the variable. Where subscript varies with respect to its definition must specify parameters value that takes. It only applies to equations of variables.

- **COMPONENTE 3**
  The COMPONENT 3 is not required for linear models.

CONSTRAINTS

The constraints required to model the problem VRP are presented in the following slides

The expression highlighted in gray corresponds to the form as the equation must be implemented in OPTEX, which implies:

1. The grouping to the left side of all the terms that contain variables; and
2. On the right side must be a parameter or a constant value. This value is located in the master table of the constraints.

Below, an example:

$$\Sigma_{k \in TRK(c\ v)} VCL_{v,c,k} = AVL_v$$

$$\Sigma_{k \in TRK(c\ v)} VCL_{v,c,k} - AVL_v = 0$$

$$\forall v \in VEH \quad \forall c \in NOV(v)$$

CONSTRAINTS

<table>
<tr><th>CONSTRAINT</th><th>DESCRIPTION- EQUATION</th><th>UNIT</th></tr>
<tr><td>$\mathbf{SANO_{v,c}}$</td><td>

Departure from the origin node

Establishes that any vehicle v used should leave the origin node c (warehouse) in which it is located.

$$\Sigma_{k \in TRK(c\ v)} VCL_{v,c,k} = AVL_v$$

$$\Sigma_{k \in TRK(c\ v)} VCL_{v,c,k} - AVL_v = 0$$

$$\forall v \in VEH \quad \forall c \in NOV(v)$$

Exists for all vehicle **v (v∈VEH)** and for the node source c in which is located the vehicle **v (c∈NOV (v))**.

Sets:
TRK(c,v) Customers **k** you can visit from **c** in vehicle **v**
VEH Vehicles **v**
NOV(v) Warehouse **c** in which is located the vehicle **v**
Variables:
$\mathbf{VCL_{v,c,k}}$ Determines if a vehicle **v** goes from customer **c** to customer **k**
$\mathbf{AVL_v}$ Determines the use of a vehicle **v**

$$\Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - AVL_v = 0$$
$$\forall v \in VEH \quad \forall c \in NOV(v)$$

SIGN	COMPONENT 1	COMPONENT 2
+	S	k/TRK
+	1	VCL
-	1	AVL

</td><td></td></tr>
</table>

CONSTRAINTS

CONSTRAINT	DESCRIPTION- EQUATION	UNIT
$\mathbf{ENSA_{v,c}}$	**Arrival and departures of a node** It establishes that any vehicle that visits to a destination should leave this $\Sigma_{k \in TRK(c,v)} VCL_{v,k,c} = \Sigma_{k \in TRK(c,v)} VCL_{v,c,k}$ $\Sigma_{k \in TRK(c\ v)} VCL_{v,k,c} - \Sigma_{k \in TRK(c\ v)} VCL_{v,c,k} = 0$ $\forall v \in VEH \quad \forall c \in NCV(v)$ Sets: **TRK(c,v)** Customers **k** you can visit from **c** in vehicle **v** **VEH** Vehicles v **NCV(v)** Customer **c** it can meet the vehicle **v** Variables: $\mathbf{VCL_{v,c,k}}$ Determines if a vehicle **v** goes from customer **c** to customer **k** $\Sigma_{k \in TRK(c,v)} VCL_{v,k,c} - \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} = 0$ $\forall v \in VEH \quad \forall c \in NCV(v)$	

SIGN	COMPONENT 1	COMPONENT 2
+	S	k/TRK
+	1	VCL
-	S	k/TRK
+	1	VKL

Given the equation relates the variable $\mathbf{VCL_{v,c,k}}$ from two points of view: flow from **c** to **k** and **k** to **c** flow, in **OPTEX** is considered necessary the inclusion of the concept of **ALIAS** to represent the variable when you have the indexes in different order, in this case the name given to the alias is $\mathbf{VKL_{v,k,c}}$ which allows to formulate the equation without having to specify the indexes.

CONSTRAINTS

<table>
<tr><th>CONSTRAINT</th><th>DESCRIPTION- EQUATION</th><th>UNIT</th></tr>
<tr><td>$UTVE_v$</td><td>
Use of vehicles
It establishes that only if the vehicle v is used can make travel between nodes c and k.

$$\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} \leq \infty \times AVL_v$$
$$\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - \infty \times AVL_v \leq 0$$
$$\forall v \in VEH$$
Exists for each vehicle v ($v \in VEH$).

Sets:
NCV(v) Customers c that can be meet with the vehicle v
TRK(c,v) Customers k you can visit from c in vehicle v
VEH Vehicles v
Variables:
$VCL_{v,c,k}$ Determines if a vehicle v goes from customer c to customer k
AVL_v Determines the use of a vehicle v

$$\Sigma_{c \in NCV(v)} \Sigma_{k \in TRK(c,v)} VCL_{v,c,k} - \infty \times AVL_v \leq 0$$
$$\forall v \in VEH$$
<table>
<tr><th>SIGN</th><th>COMPONENT 1</th><th>COMPONENT 2</th></tr>
<tr><td>+</td><td>S</td><td>c/NCV</td></tr>
<tr><td>+</td><td>S</td><td>k/TRK</td></tr>
<tr><td>+</td><td>1</td><td>VCL</td></tr>
<tr><td>-</td><td>INFI</td><td>AVL</td></tr>
</table>
</td><td></td></tr>
</table>

CONSTRAINTS

<table>
<tr><th>CONSTRAINT</th><th>DESCRIPTION- EQUATION</th><th>UNIT</th></tr>
<tr><td>$\mathbf{VCLI_c}$</td><td>

Visit of destination
Set at least one vehicle **v** visit customer **c.**

$$\Sigma_{v\in VEC(c)} \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 1$$

$$\forall c\in DEC$$

Exists for any client that should be visited **c** (**c∈NOV(v)**).

Sets:
VEC(c) Vehicles **v** that can meet the client **c**
TRK(c,v) Customers **k** you can visit from **c** in vehicle **v**
DEC Customers **c**
Variables:
$\mathbf{VCL_{v,c,k}}$ Determines if a vehicle **v** goes from customer **c** customer **k**

$$\Sigma_{v\in VEC(c)} \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 1$$
$$\forall c\in DEC$$

SIGN	COMPONENT 1	COMPONENT 2
+	S	v/VEC
+	S	k/TRK
+	1	VCL

</td><td></td></tr>
</table>

CONSTRAINTS

CONSTRAINT	DESCRIPTION- EQUATION	UNIT
$CAPP_v$	**Load capacity vehicles** Sets that demand covered by the vehicle weight cannot be greater than its capacity in weight. $$\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c\ v)} DEMP_c \times VCL_{v,c,k} \leq CAPP_v$$ $$\forall v\in VEH$$ Sets: **NCV(v)** Customers **c** that can be meet with the vehicle **v** **TRK(c,v)** Customers **k** you can visit from **c** in vehicle **v** **VEH** Vehicles Parameters: **$DEMP_c$** Weight of the customer's order **c** (kg) **$CAPP_v$** Capacity of the vehicle in weight (kg) Variables: **$VCL_{v,c,k}$** Determines if a vehicle **v** goes from customer **c** customer **k** Parameter **$DEMP_c$** parameter associated with the weight of the order to deliver client **c** is calculated as: $$DEMP_c = \Sigma_{b\in CAC(c)} NUCD_{c,b} \times PECA_b$$ Sets: **CAC(c)** Boxes (b) that are part of the order of the client **c** Parámetros: **$NUCD_{c,b}$** Amount of boxes type b should be released for customer **c** (und) **$PECA_b$** Weight boxes type **b** (kg) $$\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c\ v)} DEMP_c \times VCL_{v,c,k} \leq CAPP_v$$ $$\forall v\in VEH$$	kg

SIGN	COMPONENT 1	COMPONENT 2
+	S	c/VEC
+	S	k/TRK
+	DEMP	VCL

CONSTRAINTS

CONSTRAINT	DESCRIPTION- EQUATION	UNIT
$\mathbf{CAPV_v}$	**Volumetric capacity of vehicles**	m3

Volumetric capacity of vehicles

Sets that demand covered by the vehicle weight cannot be greater than its capacity in volume.

$$\sum_{c \in NCV(v)} \sum_{k \in TRK(c\ v)} \mathbf{DEMV_c \times VCL_{v,c,k} \leq CAPV_v}$$

$$\forall v \in VEH$$

Sets:

NCV(v) Customers **c** that can be meet with the vehicle **v**

TRK(c,v) Customers **k** you can visit from **c** in vehicle **v**

VEH Vehicles **v**

Parameters:

$\mathbf{DEMV_c}$ Order volume of customer **c** (m3)

$\mathbf{CAPV_v}$ Capacity of the vehicle **v** in volume (m3)

Variables:

$\mathbf{VCL_{v,c,k}}$ Determines if a vehicle **v** goes from customer **c** customer **k**

The parameter $\mathbf{DEMV_c}$ parameter associated with the volume of the order to deliver client **c** is calculated as:

$$\mathbf{DEMV_c} = \sum_{b \in CAC(c)} \mathbf{NUCD_{c,b} \times VOCA_b}$$

Sets:

CAC(c) Cajas **b** que hacen parte del pedido del cliente **c**

Parameters:

$\mathbf{NUCD_{c,b}}$ Cantidad de cajas tipo **b** que deben ser despachadas al cliente **c** (und)

$\mathbf{VOCA_b}$ Volume of boxes type **b** (m3)

$$\sum_{c \in NCV(v)} \sum_{k \in TRK(c\ v)} \mathbf{DEMV_c \times VCL_{v,c,k} \leq CAPV_v}$$
$$\forall v \in VEH$$

SIGN	COMPONENT 1	COMPONENT 2
+	S	c/VEC
+	S	k/TRK
+	DEMV	VCL

CONSTRAINTS

3. The third (sheet **ECUACION**) which contains the terms that break down every equation of the model.

	A	B	C	D	E	F
1	COD_RES	SEQ	SIGNO	CAMPO_1	CAMPO_2	CAMPO_3
2	Constraint Code	Sequence	Sign (+ or -)	Contains: SUM c	Contains: Subsc	Contains: Variat
3	SANO	1	+	S	k/TRK	
4	SANO	2	+	1	VCL	
5	SANO	3	-	1	AVL	
6	ENSA	1	+	S	k/TRK	
7	ENSA	2	+	1	VCL	
8	ENSA	3	-	S	k/TRK	
9	ENSA	4	+	1	VKL	
10	UTVE	1	+	S	c/NCV	
11	UTVE	2	+	S	k/TRK	
12	UTVE	3	+	1	VCL	
13	UTVE	4	-	INFI	AVL	
14	VCLI	1	+	S	v/VEC	
15	VCLI	2	+	S	k/TRK	
16	VCLI	3	+	1	VCL	
17	CAPP	1	+	S	c/VEC	
18	CAPP	2	+	S	k/TRK	
19	CAPP	3	+	DEMP	VCL	
20	CAPV	1	+	S	c/VEC	
21	CAPV	2	+	S	k/TRK	
22	CAPV	3	+	DEMV	VCL	

$$\Sigma_{k\in TRK(c,v)} VCL_{v,k,c} - \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 0$$

$$\Sigma_{k\in TRK(c,v)} VCL_{v,k,c} - \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 0$$

$$\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} - \infty \times AVL_v \leq 0$$

$$\Sigma_{v\in VEC(c)} \Sigma_{k\in TRK(c,v)} VCL_{v,c,k} = 1$$

$$\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c\,v)} DEMP_c \times VCL_{v,c,k} \leq CAPP_v$$

$$\Sigma_{c\in NCV(v)} \Sigma_{k\in TRK(c\,v)} DEMV_c \times VCL_{v,c,k} \leq CAPV_v$$

VARIABLES ALIAS

Given that the $ENSA_{c,v}$ equation, relates the variable $VCL_{v,c,k}$ from two points of view: flow from c to k and flow from k to c, requires considered the inclusion of the concept of ALIAS to represent the variable when it is necessary to have the indexes in different order.

Below, the variables alias required:

VARIABLES			
VARIABLE ALIAS	**DESCRIPTION**	**VARIABLE**	**EXISTENCE CONDITIONS**
$VKL_{v,k,c}$	Binary variable alias for $VCL_{v,k,c}$ which determines if the vehicle goes from the origin **k** node to destination node **c** Exists for every vehicle v considered in the problem, every customer k which can be attended by the vehicle v **k∈NKV(v)**) and by every node **c** which can be visited from the node **c** in vehicle **v** (**k∈TRC(k,v)**)	$VCL_{v,c,k}$	∀v∈VEH ∀k∈NKV(v) ∀c∈TRC(k,v)

VARIABLES ALIAS

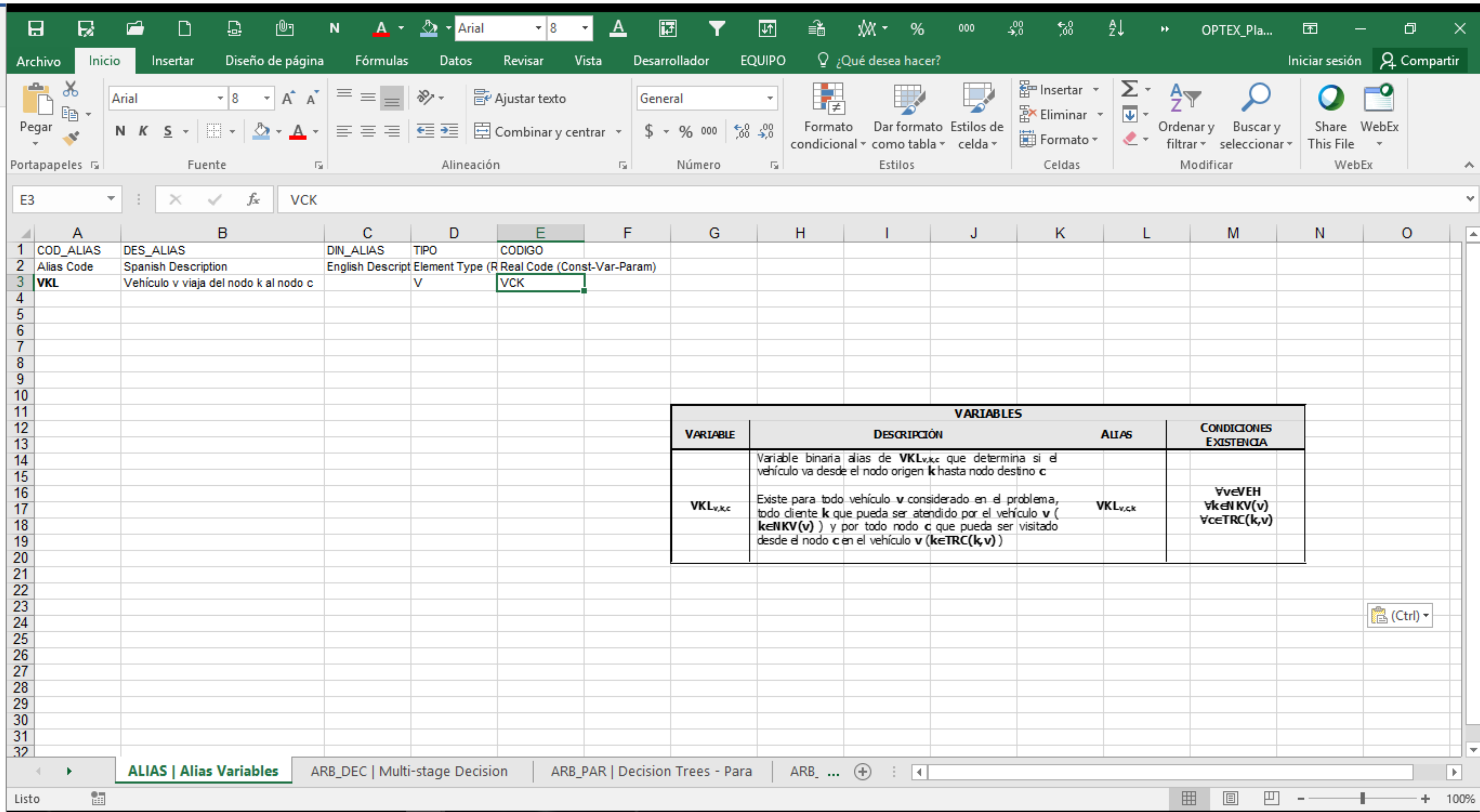
The definition of the variables alias implies filling two tables:

1. **The first (sheet ALIAS) which determines the general attributes of the variable alias**
2. **The second (sheet IND_ALIA) to define the indexes of variables alias and their conditions of existence.**

VARIABLES			
VARIABLE ALIAS	**DESCRIPTION**	**VARIABLE**	**EXISTENCE CONDITIONS**
$VKL_{v,k,c}$	Binary variable alias for **$VCL_{v,k,c}$** which determines if the vehicle goes from the origin **k** node to destination node **c** Exists for every vehicle v considered in the problem, every customer k which can be attended by the vehicle v **$k \in NKV(v)$**) and by every node **c** which can be visited from the node **c** in vehicle **v** (**$k \in TRC(k,v)$**)	**$VCL_{v,c,k}$**	**$\forall v \in VEH$** **$\forall k \in NKV(v)$** **$\forall c \in TRC(k,v)$**

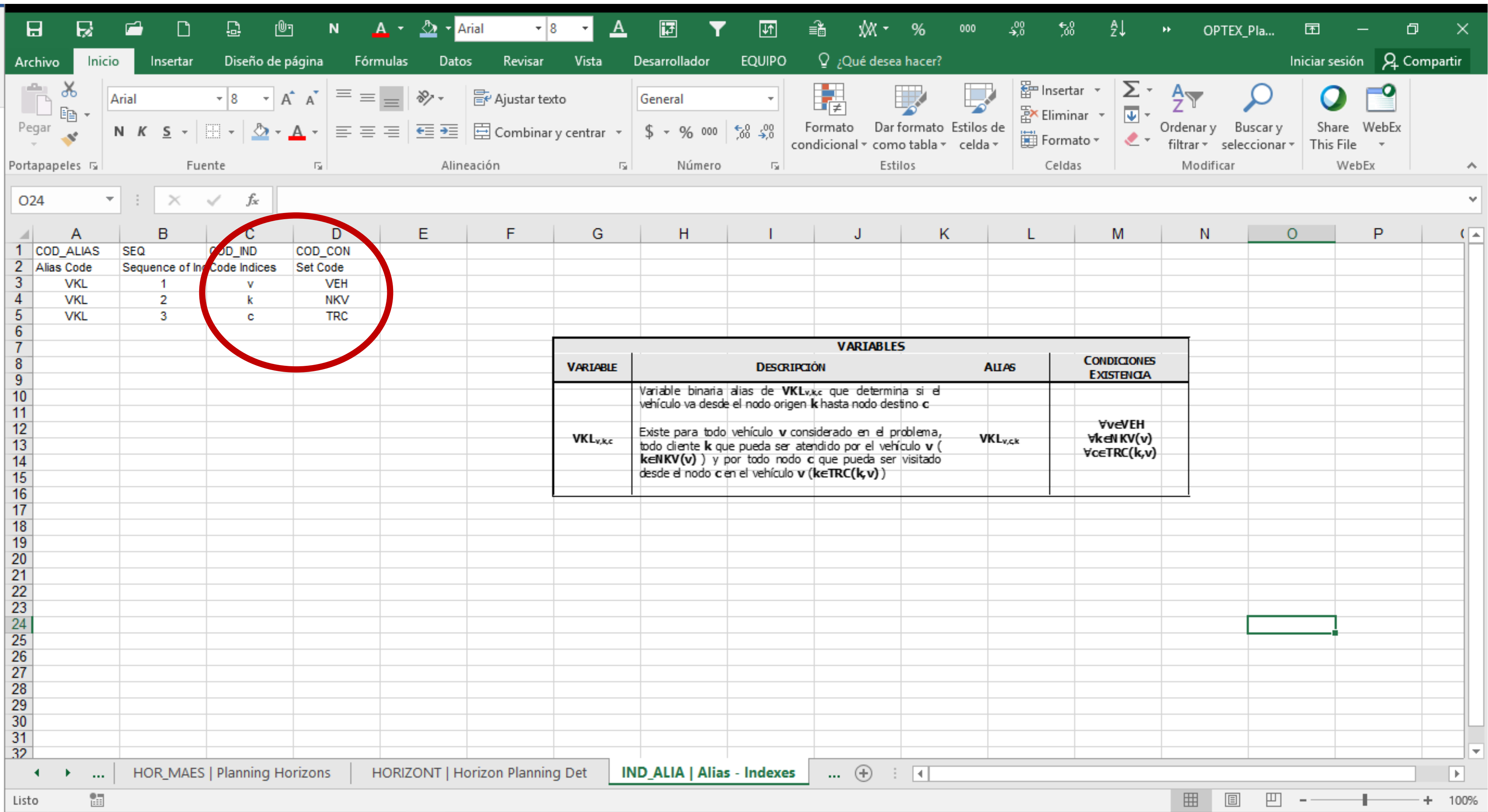
VARIABLES ALIAS

1. **The first (sheet ALIAS) which determines the general attributes of the variable**



VARIABLES ALIAS

2. The second (sheet **IND_ALIA**) to define the indexes of variables and their **conditions of existence**.



OBJECTIVE FUNCTIONS

The objective of the optimization in the VRP model can be of different types according to the criteria of the planner. In this case is the minimization of the total operation costs that are integrated by the fixed costs of use of vehicles plus vehicles travel costs.

The implementation of the equations related to the objective function requires three tables/sheets:

1. **The first (FUNOBJ) which determines the general attributes of the objective functions. The template, which is presented below, includes the classification of the objective function type: SIM, which implies that it is a basic equation that sum the variables multiplied by its corresponding cost and MUL involving the weighted sum of objective functions, single or multiple.**

2. **The second (VAR_OBJ) that defines the relationships between variables and their costs in objective function, including the sign in the sum.**

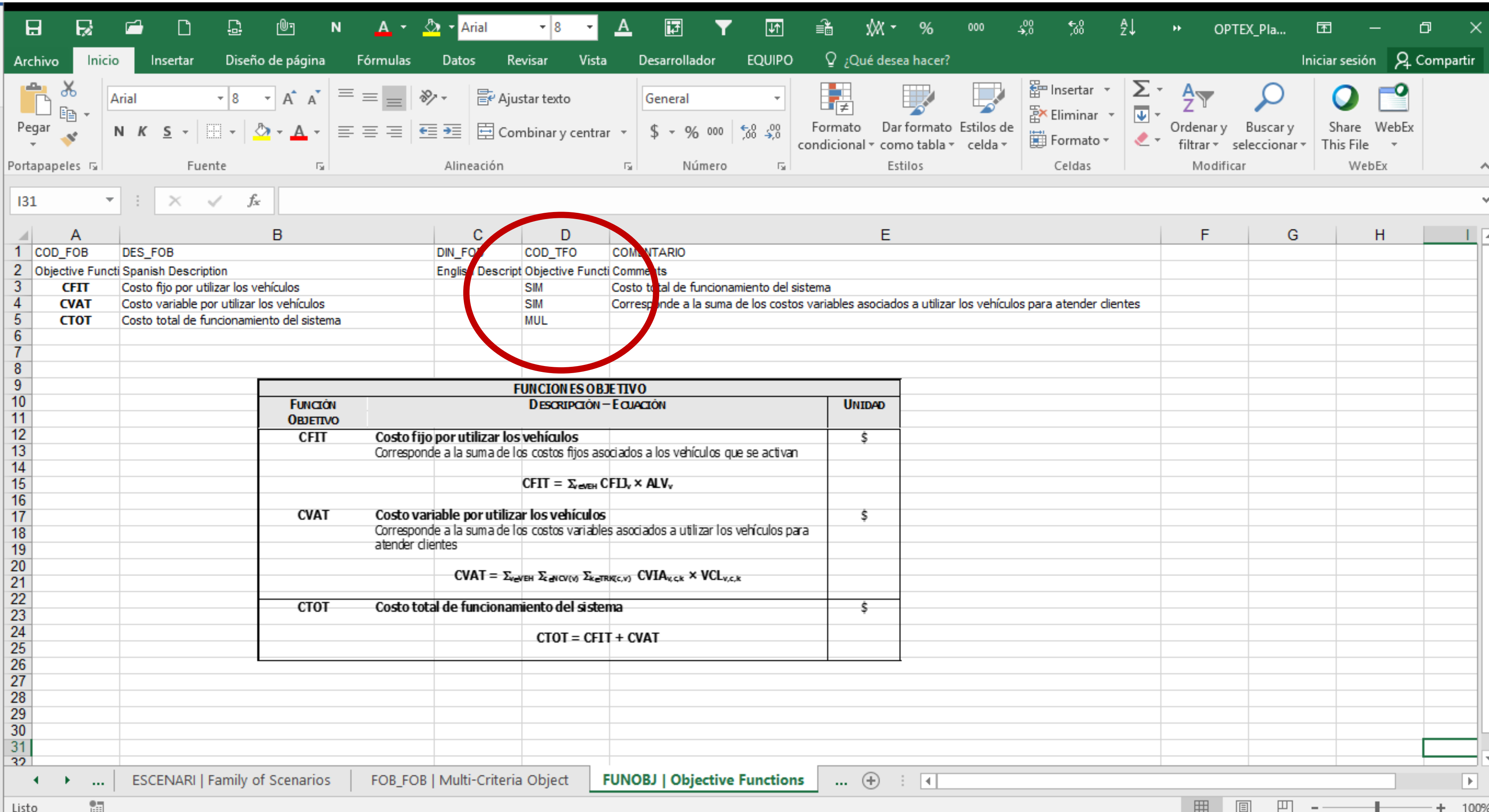
3. **The third (FOB_FOB) that define the relationship between objective functions, including a weighting factor, which in conventional objective functions is equal to 1.**

OBJECTIVE FUNCTIONS

OBJECTIVE FUNCTIONS		
OBJECTIVE FUNCTION	**DESCRIPTION - EQUATION**	**UNIT**
CFIT	**Fixed cost for used vehicles** It corresponds to the sum of the fixed costs associated with vehicles that are activated $\mathbf{CFIT = \Sigma_{v \in VEH}\ CFIJ_v \times AVL_v}$ Sets: **VEH** Vehicles **v** Parámetros: $\mathbf{CFIJ_v}$ Fixed cost of vehicle **v** Variables: $\mathbf{AVL_v}$ Determines the use of a vehicle **v**	$
CVAT	**Variable cost for used vehicles** It corresponds to the sum of the variable costs to use vehicles to meet customers $\mathbf{CVAT = \Sigma_{v \in VEH}\ \Sigma_{c \in NCV(v)}\ \Sigma_{k \in TRK(c,v)}\ CVIA_{v,c,k} \times VCL_{v,c,k}}$ Sets: **NCV(v)** Customers **c** that can be meet with the vehicle **v** **TRK(c,v)** Customers **k** you can visit from **c** in vehicle **v** **VEH** Vehicles **v** Parameters: $\mathbf{CVIA_{v,c,k}}$ Cost of meet a client k from the node c with the vehicle **v** ($) Variables: $\mathbf{VCL_{v,c,k}}$ Determines if a vehicle v goes from customer c customer **k** The cost-per-trip $\mathbf{CVIA_{v,c,k}}$, is calculated as: $\mathbf{CVIA_{v,c,k} = COVA_v \times DIST_{c,k}}$ $\mathbf{COVA_v}$ Cost per kilometer of vehicle **v** ($/kmt) $\mathbf{DIST_{c,k}}$ Distance between the node c and the node **k** (kmt) COVAv Cost per kilometer of vehicle v ($/kmt) DISTc, Distteance between the node c and the node k (kmt)	$
CTOT	**Total cost of operation of the system** $\mathbf{CTOT = CFIT + CVAT}$	$

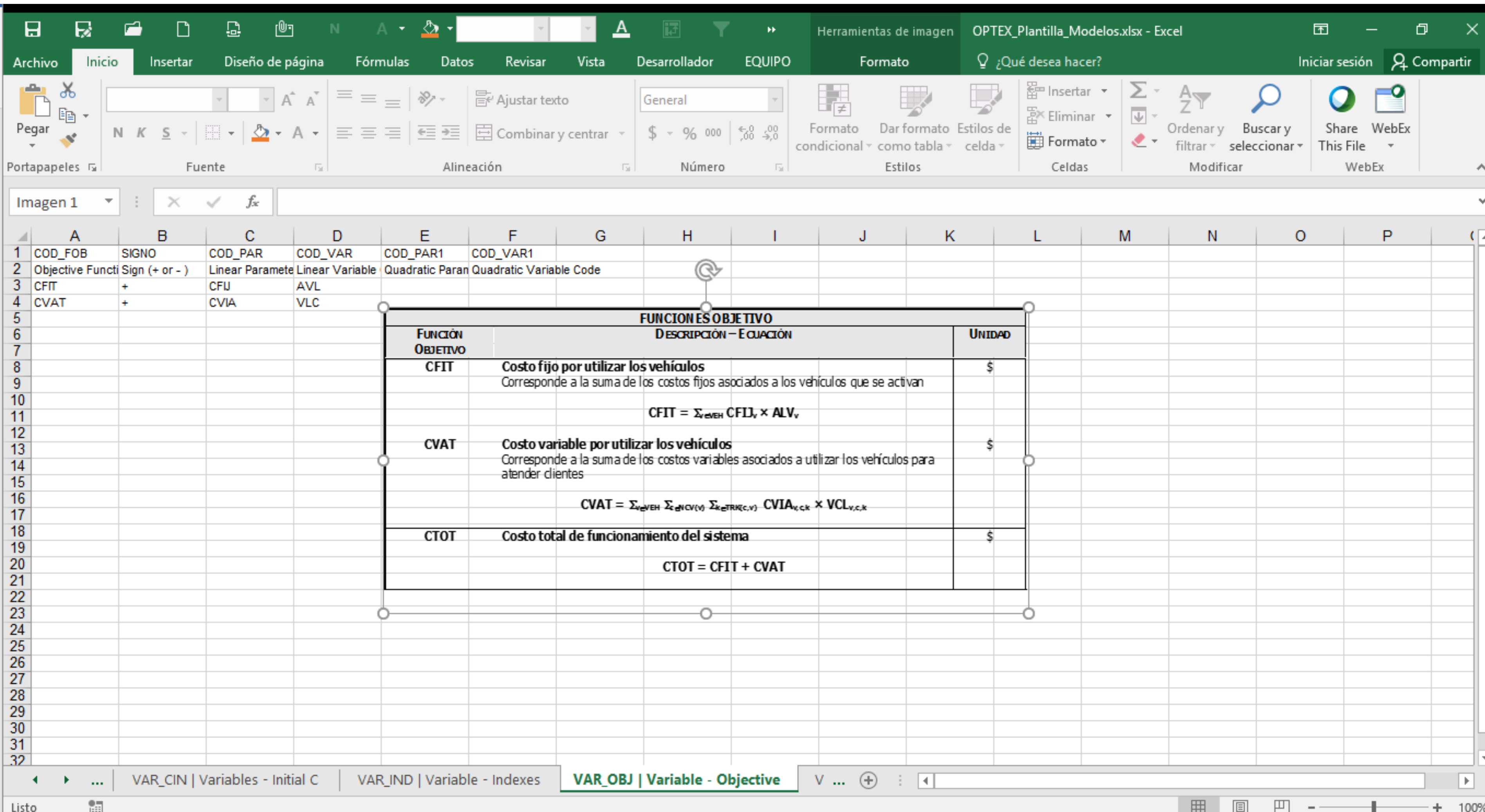
OBJECTIVE FUNCTIONS

1. The first (FUNOBJ) which determines the general attributes of the objective functions; includes the classification of the objective function type: **SIM**, which implies that it is a basic equation that sum the variables multiplied by its corresponding cost and **MUL** involving the weighted sum of objective functions.



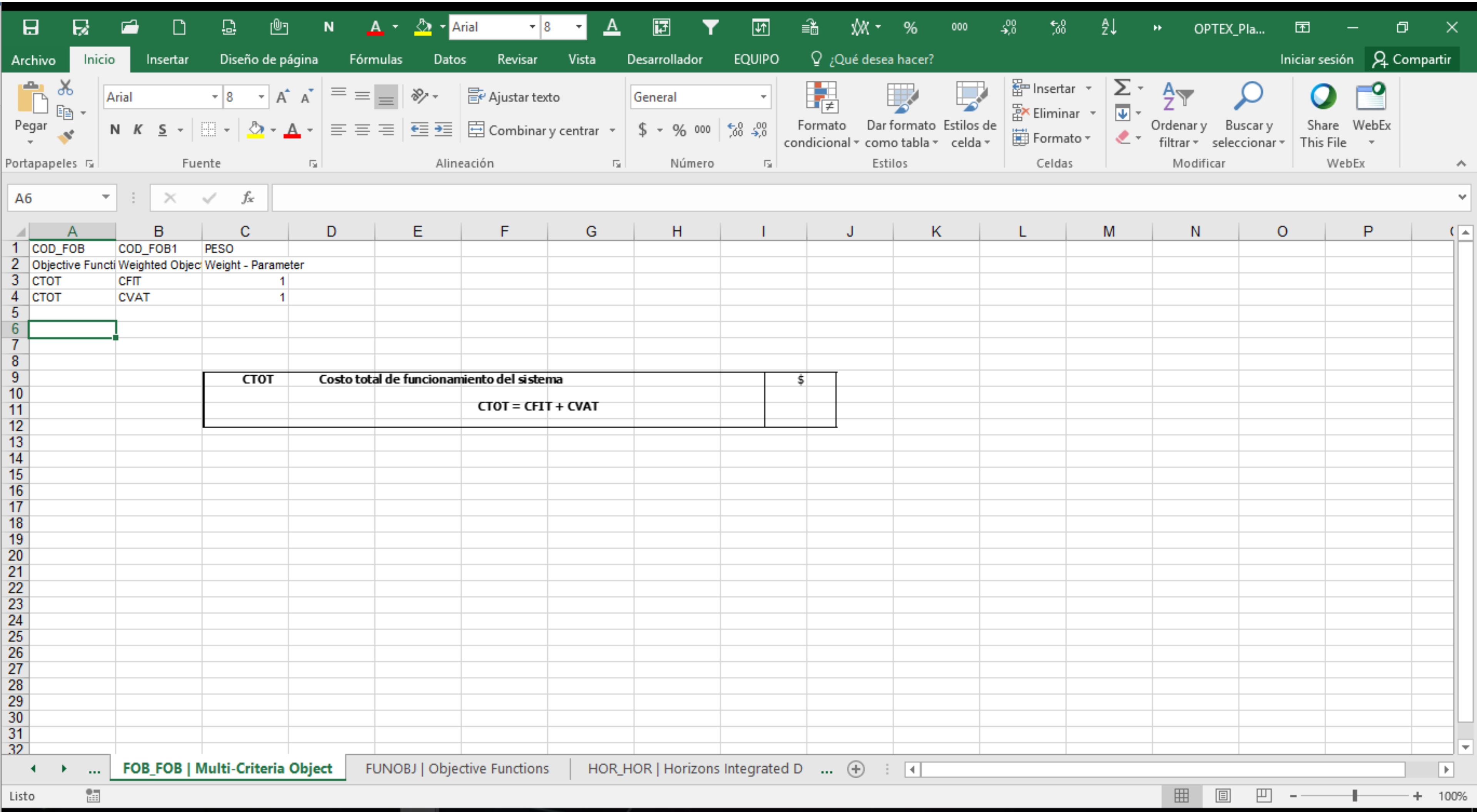
OBJECTIVE FUNCTIONS

2. The second (VAR_OBJ) that defines the relationships between variables and their costs in objective function, including the sign in the sum.



OBJECTIVE FUNCTIONS

3. **The third (FOB_FOB) that define the relationship between objective functions, including a weighting factor, which in conventional objective functions is equal to 1.**



PARAMETERS

The parameters of the models must configure, associating them with a name/code that is used in the equations of the mathematical model. The value of a parameter can be set in two ways:

- **From the data contained in the information system tables; and**
- **As a result of the evaluation of an equation that involves other parameters, so that the user does not have to perform manual calculations.**

The parameters read are presented in the following table, including the table and the field that contains the numeric data.

BASIC PARAMETERS				
PARAMETERS	**DESCRIPTION**	**UNIT**	**TABLE**	**FIELD**
$CAPV_v$	**Capacity of the vehicle in volume** Volumetric capacity of the vehicle, measured in cubic metres	m3	VEHICULOS	CAPV
$COVA_v$	**Variable costs for using a vehicle v** Cost per kilometer by using the vehicle	$/kmt	VEHICULOS	COVA
$CFIJ_v$	**Fixed costs for using a vehicle v** Fixed cost of use vehicle v	$/día	VEHICULOS	CUVE
$DIST_{c,k}$	**Distance nodes** Distance between the origin node and the destination node	km	NOD_NOD	DIST
$NUCA_{w,b}$	**Number of boxes of order** Number of boxes order that must be transported to the node	und	PED_CAJ	NUCA
$PECA_b$	**Weight box** Weight in kg of boxes	kg	CAJAS	PECA
$VOCA_b$	**Volume of boxes** Volume of the boxes in cubic meters	m3	CAJAS	VOCA

PARAMETERS

The calculated parameters are summarized in the following table:

CALCULATED PARAMETERS		
PARAMETER	**DESCRIPTION**	**UNIT**
$\mathbf{CVIA_{v,c,k}}$	**Cost of travel between nodes** Cost of travel of the vehicle from origin node to the destination node $\mathbf{CVIA_{v,c,k}} = \mathbf{COVA_v} \times \mathbf{DIST_{c,k}}$ Parameters: $\mathbf{COVA_v}$ Cost per kilometer of vehicle **v** ($/kmt) $\mathbf{DIST_{c,k}}$ Distance between the node c and the node **k** (kmt)	$
$\mathbf{DEMP_c}$	**Demand in weight** The node demand expressed in kilograms $\mathbf{DEMP_c} = \Sigma_{\mathbf{b \in CAC(c)}} \mathbf{NUCD_{c,b}} \times \mathbf{PECA_b}$ Sets: **CAC(c)** Boxes (b) that part of the order of the client **c** Parameters: $\mathbf{NUCD_{c,b}}$ Amount of boxes type **b** should be released for customer **c** (und) $\mathbf{PECA_b}$ Weight boxes type **b** (kg)	kg
$\mathbf{DEMV_c}$	**Demand in volume** Demand for the node, expressed as a volume $\mathbf{DEMV_c} = \Sigma_{\mathbf{b \in CAC(c)}} \mathbf{NUCD_{c,b}} \times \mathbf{VOCA_b}$ Sets: **CAC(c)** Boxes (b) that part of the order of the client **c** Parameters: $\mathbf{NUCD_{c,b}}$ Amount of boxes type **b** should be released for customer **c** (und) $\mathbf{VOCA_b}$ Volume of boxes type **b** (m3)	m3

PARAMETERS

The definition of the parameters implies filling three tables:

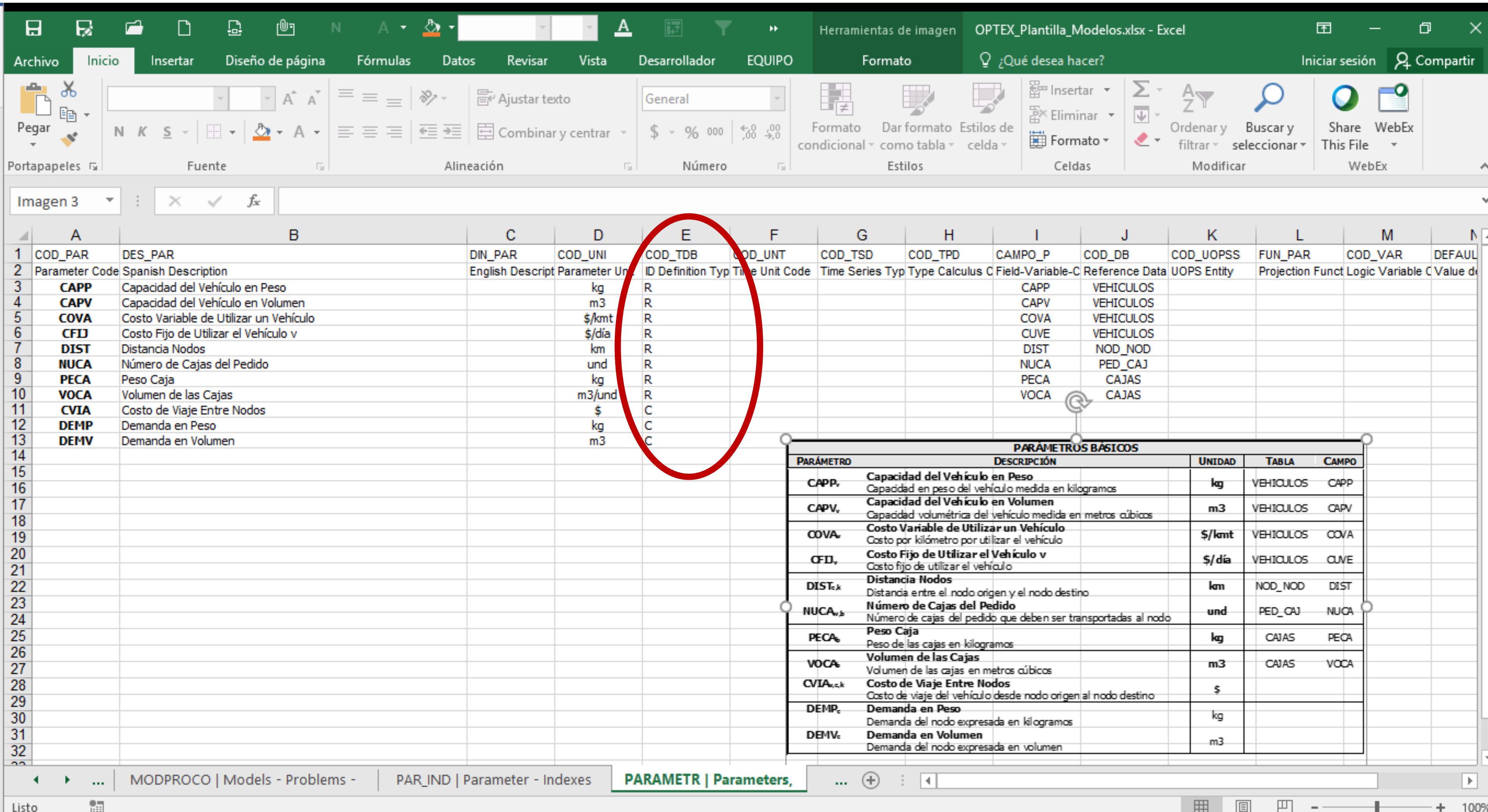
1. **The first (sheet PARAMETR) which determines the general attributes of the parameters**

2. **The second (sheet PAR_IND) to define the indexes of the parameters.**

3. **The third (sheet ECUACION) which contains the terms that break down every equation of the model.**

PARAMETERS

1. The first (sheet **PARAMETR**) which determines the general attributes of the parameters- It includes the type (field **COD_TDB**): **R** Read parameter and **C** calculated parameter



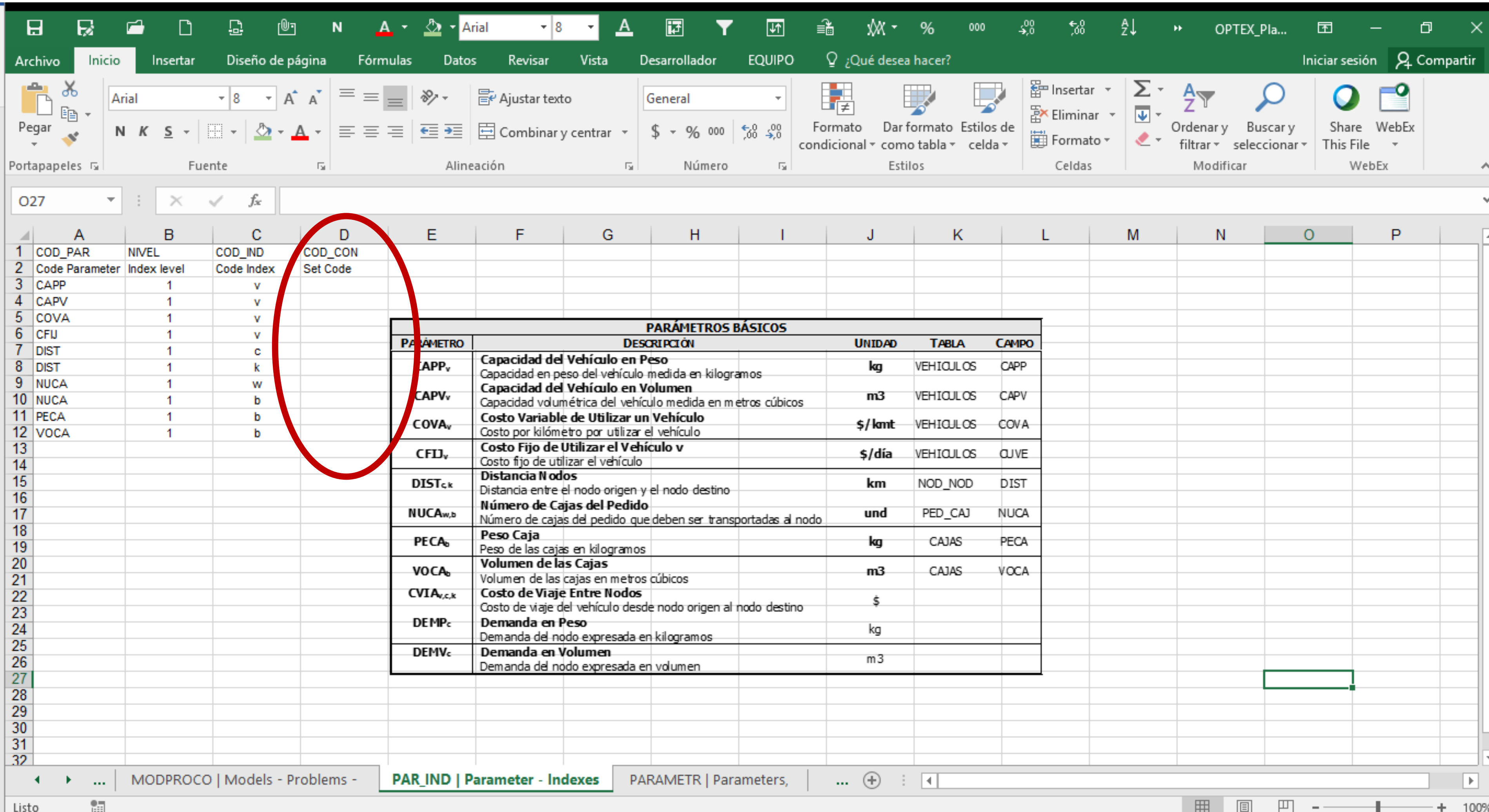
COD_PAR	DES_PAR	DIN_PAR	COD_UNI	COD_TDB	COD_UNT	COD_TSD	COD_TPD	CAMPO_P	COD_DB	COD_UOPSS	FUN_PAR	COD_VAR	DEFAUL
Parameter Code	Spanish Description	English Descript	Parameter Un	ID Definition Typ	Time Unit Code	Time Series Typ	Type Calculus C	Field-Variable-C	Reference Data	UOPS Entity	Projection Funct	Logic Variable C	Value d
CAPP	Capacidad del Vehículo en Peso		kg	R				CAPP	VEHICULOS				
CAPV	Capacidad del Vehículo en Volumen		m3	R				CAPV	VEHICULOS				
COVA	Costo Variable de Utilizar un Vehículo		$/kmt	R				COVA	VEHICULOS				
CFIJ	Costo Fijo de Utilizar el Vehículo v		$/día	R				CUVE	VEHICULOS				
DIST	Distancia Nodos		km	R				DIST	NOD_NOD				
NUCA	Número de Cajas del Pedido		und	R				NUCA	PED_CAJ				
PECA	Peso Caja		kg	R				PECA	CAJAS				
VOCA	Volumen de las Cajas		m3/und	R				VOCA	CAJAS				
CVIA	Costo de Viaje Entre Nodos		$	C									
DEMP	Demanda en Peso		kg	C									
DEMV	Demanda en Volumen		m3	C									

PARÁMETROS BÁSICOS

PARÁMETRO	DESCRIPCIÓN	UNIDAD	TABLA	CAMPO
$CAPP_v$	**Capacidad del Vehículo en Peso** Capacidad en peso del vehículo medida en kilogramos	kg	VEHICULOS	CAPP
$CAPV_v$	**Capacidad del Vehículo en Volumen** Capacidad volumétrica del vehículo medida en metros cúbicos	m3	VEHICULOS	CAPV
$COVA_v$	**Costo Variable de Utilizar un Vehículo** Costo por kilómetro por utilizar el vehículo	$/kmt	VEHICULOS	COVA
$CFIJ_v$	**Costo Fijo de Utilizar el Vehículo v** Costo fijo de utilizar el vehículo	$/día	VEHICULOS	CUVE
$DIST_{c,k}$	**Distancia Nodos** Distancia entre el nodo origen y el nodo destino	km	NOD_NOD	DIST
$NUCA_{w,b}$	**Número de Cajas del Pedido** Número de cajas del pedido que deben ser transportadas al nodo	und	PED_CAJ	NUCA
$PECA_b$	**Peso Caja** Peso de las cajas en kilogramos	kg	CAJAS	PECA
$VOCA_b$	**Volumen de las Cajas** Volumen de las cajas en metros cúbicos	m3	CAJAS	VOCA
$CVIA_{v,c,k}$	**Costo de Viaje Entre Nodos** Costo de viaje del vehículo desde nodo origen al nodo destino	$		
$DEMP_c$	**Demanda en Peso** Demanda del nodo expresada en kilogramos	kg		
$DEMV_c$	**Demanda en Volumen** Demanda del nodo expresada en volumen	m3		

PARAMETERS

2. **The second (sheet PAR_IND) to define the indexes of the parameters. It isn't necessary to fill the field COD_CON associated to set.**

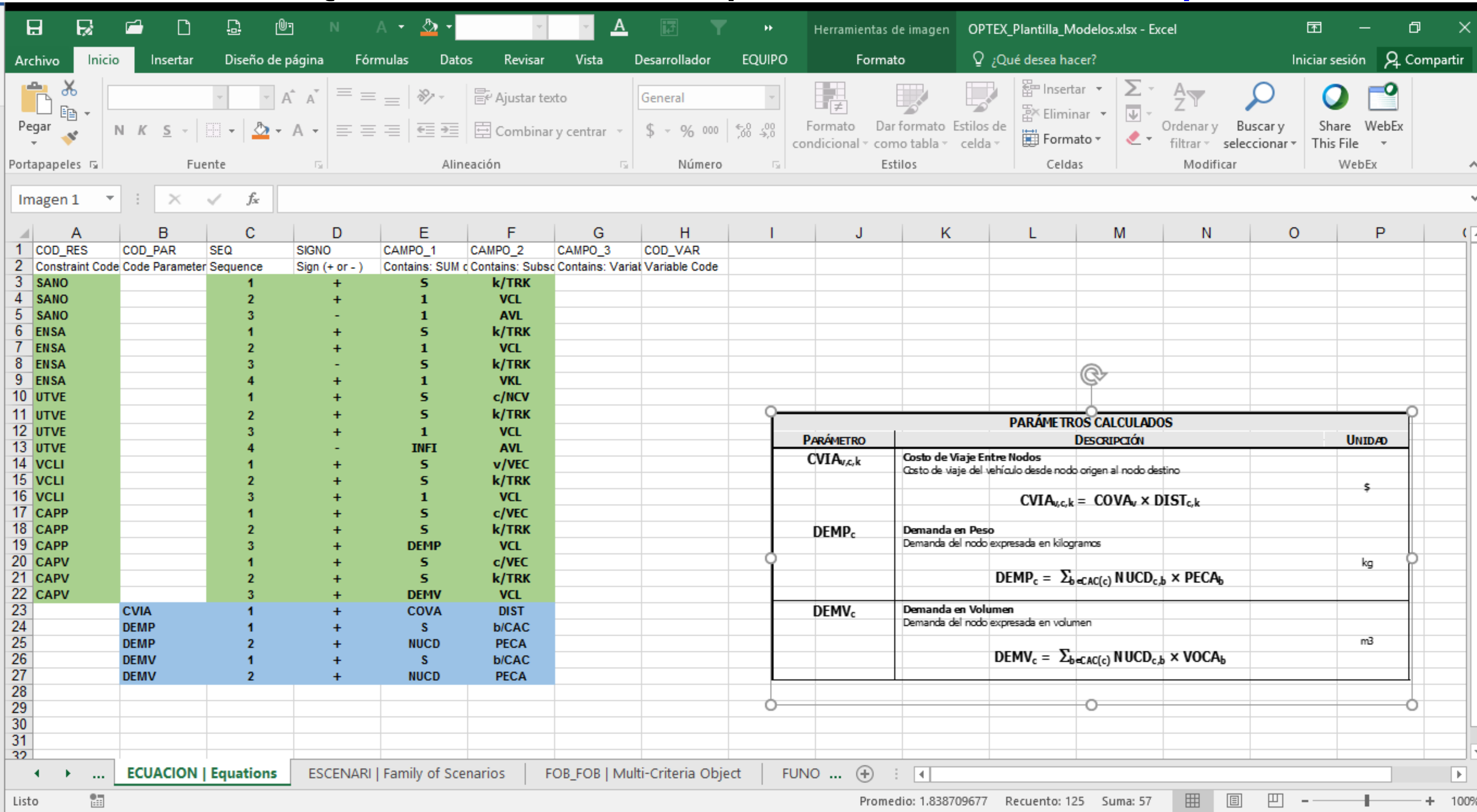


A	B	C	D
COD_PAR	NIVEL	COD_IND	COD_CON
Code Parameter	Index level	Code Index	Set Code
CAPP	1	v	
CAPV	1	v	
COVA	1	v	
CFIJ	1	v	
DIST	1	c	
DIST	1	k	
NUCA	1	w	
NUCA	1	b	
PECA	1	b	
VOCA	1	b	

PARÁMETROS BÁSICOS				
PARÁMETRO	DESCRIPCIÓN	UNIDAD	TABLA	CAMPO
$CAPP_v$	**Capacidad del Vehículo en Peso** Capacidad en peso del vehículo medida en kilogramos	kg	VEHICULOS	CAPP
$CAPV_v$	**Capacidad del Vehículo en Volumen** Capacidad volumétrica del vehículo medida en metros cúbicos	m3	VEHICULOS	CAPV
$COVA_v$	**Costo Variable de Utilizar un Vehículo** Costo por kilómetro por utilizar el vehículo	$/kmt	VEHICULOS	COVA
$CFIJ_v$	**Costo Fijo de Utilizar el Vehículo v** Costo fijo de utilizar el vehículo	$/día	VEHICULOS	CUVE
$DIST_{c,k}$	**Distancia Nodos** Distancia entre el nodo origen y el nodo destino	km	NOD_NOD	DIST
$NUCA_{w,b}$	**Número de Cajas del Pedido** Número de cajas del pedido que deben ser transportadas al nodo	und	PED_CAJ	NUCA
$PECA_b$	**Peso Caja** Peso de las cajas en kilogramos	kg	CAJAS	PECA
$VOCA_b$	**Volumen de las Cajas** Volumen de las cajas en metros cúbicos	m3	CAJAS	VOCA
$CVIA_{v,c,k}$	**Costo de Viaje Entre Nodos** Costo de viaje del vehículo desde nodo origen al nodo destino	$		
$DEMP_c$	**Demanda en Peso** Demanda del nodo expresada en kilogramos	kg		
$DEMV_c$	**Demanda en Volumen** Demanda del nodo expresada en volumen	m3		

PARAMETERS

3. The third sheet (**ECUACION**) contains the terms that break down every equation of the parameters of the model. Algebraic language is the same as described previously for the restrictions. EQUATION table stores the equations of **constraints** and the **parameters**.



COD_RES	COD_PAR	SEQ	SIGNO	CAMPO_1	CAMPO_2	CAMPO_3	COD_VAR
Constraint Code	Code Parameter	Sequence	Sign (+ or -)	Contains: SUM c	Contains: Subsc	Contains: Variat	Variable Code
SANO		1	+	S	k/TRK		
SANO		2	+	1	VCL		
SANO		3	-	1	AVL		
ENSA		1	+	S	k/TRK		
ENSA		2	+	1	VCL		
ENSA		3	-	S	k/TRK		
ENSA		4	+	1	VKL		
UTVE		1	+	S	c/NCV		
UTVE		2	+	S	k/TRK		
UTVE		3	+	1	VCL		
UTVE		4	-	INFI	AVL		
VCLI		1	+	S	v/VEC		
VCLI		2	+	S	k/TRK		
VCLI		3	+	1	VCL		
CAPP		1	+	S	c/VEC		
CAPP		2	+	S	k/TRK		
CAPP		3	+	DEMP	VCL		
CAPV		1	+	S	c/VEC		
CAPV		2	+	S	k/TRK		
CAPV		3	+	DEMV	VCL		
	CVIA	1	+	COVA	DIST		
	DEMP	1	+	S	b/CAC		
	DEMP	2	+	NUCD	PECA		
	DEMV	1	+	S	b/CAC		
	DEMV	2	+	NUCD	PECA		

PARÁMETROS CALCULADOS

PARÁMETRO	DESCRIPCIÓN	UNIDAD
$CVIA_{v,c,k}$	**Costo de Viaje Entre Nodos** Costo de viaje del vehículo desde nodo origen al nodo destino $CVIA_{v,c,k} = COVA_v \times DIST_{c,k}$	\$
$DEMP_c$	**Demanda en Peso** Demanda del nodo expresada en kilogramos $DEMP_c = \sum_{b \in CAC(c)} NUCD_{c,b} \times PECA_b$	kg
$DEMV_c$	**Demanda en Volumen** Demanda del nodo expresada en volumen $DEMV_c = \sum_{b \in CAC(c)} NUCD_{c,b} \times VOCA_b$	m3

SETS

The sets determine the conditions of existence of the variables, the constraints and the problems, as well as the limits of the sums that make part of the equations of formulas for calculation of parameters and restrictions. The sets determine the topology of the system that is modeling.

Two types of sets are:

- **PRIMARY SETS: Directly defined from the data contained in the tables of information system; and**

- **SECONDARY SETS: They result from operations between sets.**

The sets contain elements whose class is defined by the so-called dependent index that can be indexed based on the value of other indexes that act as independent indexes.

The following slide present the basic and calculated sets employed in models. In the definition of the set is presented in parentheses the independent(s) index(es) and previously the dependent index, which determines the type of the entities/objects that contains the set.

SETS

In the definition of a read set it is necessary to fill the table and the field that contains the parameter, and, in some cases, the filter to applied to the table to select the element of the set

BASIC SETS				
SETS	**DESCRIPTION**	**TABLE**	**ELEMENT FIELD**	**FILTER**
b∈CAP(w)	**Cajas - > Pedido** Cajas que se utilizan para el almacenamiento y transporte de la mercancía.	PED_CAJ	COD_CAJ	
c∈DEC	**Destinos c** Nodos a los cuales se despacha la mercancía.	NODOS	COD_NOD	TIPO=DES
c∈NCV(v)	**Nodos c <- Vehículos** Nodos a los cuales pueden ir el vehículo para atender pedidos.	VEH_NOD	COD_NOD	
c∈NOC(k)	**Nodo Origen -> Nodo Destino** Nodos origen que se pueden conectar con el nodo destino.	NOD_NOD	COD_NOD	DIST<30
c∈NOD	**Nodos** Nodos a los cuales se les debe prestar un servicio de carga y/o descarga mercancías	NODOS	COD_NOD	
c∈NOV(v)	**Nodo Origen <- Vehículos** Nodo origen en el que está ubicado el vehículo.	NOR_VEH	COD_NOD	
k∈DEK	**Destinos k** Nodos destino a los cuales se despacha la mercancía.	NODOS	COD_NOD	TIPO=DES
k∈NKV(v)	**Nodes k <- Vehículos** Destination nodes which can go the vehicle to meet orders.	VEH_NOD	COD_NOD	
k∈NOK(c)	**Destination Node k -> Origin Node c** Nodos destino que se conectan con un nodo origen. Origin nodes connected with a destination node.	NOD_NOD	COD_NOD1	DIST<30
v∈VEH	**Vehicle** Used vehicles to collect and distribute the merchandise from and to clients	VEHICULOS	COD_VEH	
v∈VEK(k)	**Vehicles -> Destination Nodes k** Vehicles that can go to destination node to meet orders	VEH_NOD	COD_VEH	
w∈PEC(c)	**Order -> Customer** Order that can ship to the node	PEDIDOS	COD_PED	
v∈VEC(c)	**Vehicles -> Nodes** Vehicles that can go to the node to meet orders	T(NCV/c)	COD_VEH	

SETS

In the definition of a calculated set, it is necessary to define the operation that generates the set; normally an equation for a set includes two sets, but, for some operations, is possible to formulate a chain of operations, of the same type, between multiples sets. It is the case of intersections and of union of sets.

The formulation of VRP model only consider two operations: intersection (I) and super-union (S). But, OPTEX has implemented multiples type of operations that can be study in the Administrator Manual of OPTEX.

CALCULATED SETS		
SET	**DESCRIPTION**	**OPERATION**
b∈CAC(c)	**Boxes that must be transported to the node** Boxes that must be transported to the node.	$SU_{w \in PEC(c)}$ CAP(w)
c∈TRC(k,v)	**Roads on which vehicle can travel** Nodes from which you can go to the destination node using the vehicle.	NCV(v) ∩ NOC(k)
k∈DKC(c)	**Destinations k -> Destination c** Nodes from which you can go (connected) to another node	DEK ∩ NOK(c)
k∈TKD(c,v)	**Roads on which can move the vehicle (destinations)** Destination nodes from which you can go to the node using the vehicle.	TRK(c,v) ∩ DEK
k∈TRK(c,v)	**Roads on which vehicle can travel** Destination nodes from which you can go to the node using the vehicle.	NKV(v) ∩ NOK(c)
v∈VET(c,k)	**Vehicles that can travel on the road** Vehicles that can go from the origin node to the destination node.	VEK(k) ∩ VEC(c)

SETS

In the definition of a calculated set, it is necessary to define the operation that generates the set; normally an equation for a set includes two sets, but, for some operations, is possible to formulate a chain of operations, of the same type, between multiples sets. It is the case of intersections and of union of sets.

The formulation of VRP model only consider two operations: intersection (I) and super-union (S). But, OPTEX has implemented multiples type of operations that can be study in the Administrator Manual of OPTEX.

CALCULATED SETS		
SET	**DESCRIPTION**	**OPERATION**
b∈CAC(c)	**Boxes that must be transported to the node** Boxes that must be transported to the node.	$\mathbf{SU}_{w \in PEC(c)}$ **CAP(w)**
c∈TRC(k,v)	**Roads on which vehicle can travel** Nodes from which you can go to the destination node using the vehicle.	**NCV(v) ∩ NOC(k)**
k∈DKC(c)	**Destinations k -> Destination c** Nodes from which you can go (connected) to another node	**DEK ∩ NOK(c)**
k∈TKD(c,v)	**Roads on which can move the vehicle (destinations)** Destination nodes from which you can go to the node using the vehicle.	**TRK(c,v) ∩ DEK**
k∈TRK(c,v)	**Roads on which vehicle can travel** Destination nodes from which you can go to the node using the vehicle.	**NKV(v) ∩ NOK(c)**
v∈VET(c,k)	**Vehicles that can travel on the road** Vehicles that can go from the origin node to the destination node.	**VEK(k) ∩ VEC(c)**

SETS

In the definition of a calculated set, it is necessary to define the operation that generates the set; normally an equation for a set includes two sets, but, for some operations, is possible to formulate a chain of operations, of the same type, between multiples sets. It is the case of intersections and of union of sets.

The formulation of VRP model only consider two operations: intersection (I) and super-union (S). But, OPTEX has implemented multiples type of operations that can be study in the Administrator Manual of OPTEX.

CALCULATED SETS		
SET	DESCRIPTION	OPERATION
b∈CAC(c)	**Boxes that must be transported to the node** Boxes that must be transported to the node.	$SU_{w \in PEC(c)}$ CAP(w)
c∈TRC(k,v)	**Roads on which vehicle can travel** Nodes from which you can go to the destination node using the vehicle.	NCV(v) ∩ NOC(k)
k∈DKC(c)	**Destinations k -> Destination c** Nodes from which you can go (connected) to another node	DEK ∩ NOK(c)
k∈TKD(c,v)	**Roads on which can move the vehicle (destinations)** Destination nodes from which you can go to the node using the vehicle.	TRK(c,v) ∩ DEK
k∈TRK(c,v)	**Roads on which vehicle can travel** Destination nodes from which you can go to the node using the vehicle.	NKV(v) ∩ NOK(c)
v∈VET(c,k)	**Vehicles that can travel on the road** Vehicles that can go from the origin node to the destination node.	VEK(k) ∩ VEC(c)

SETS

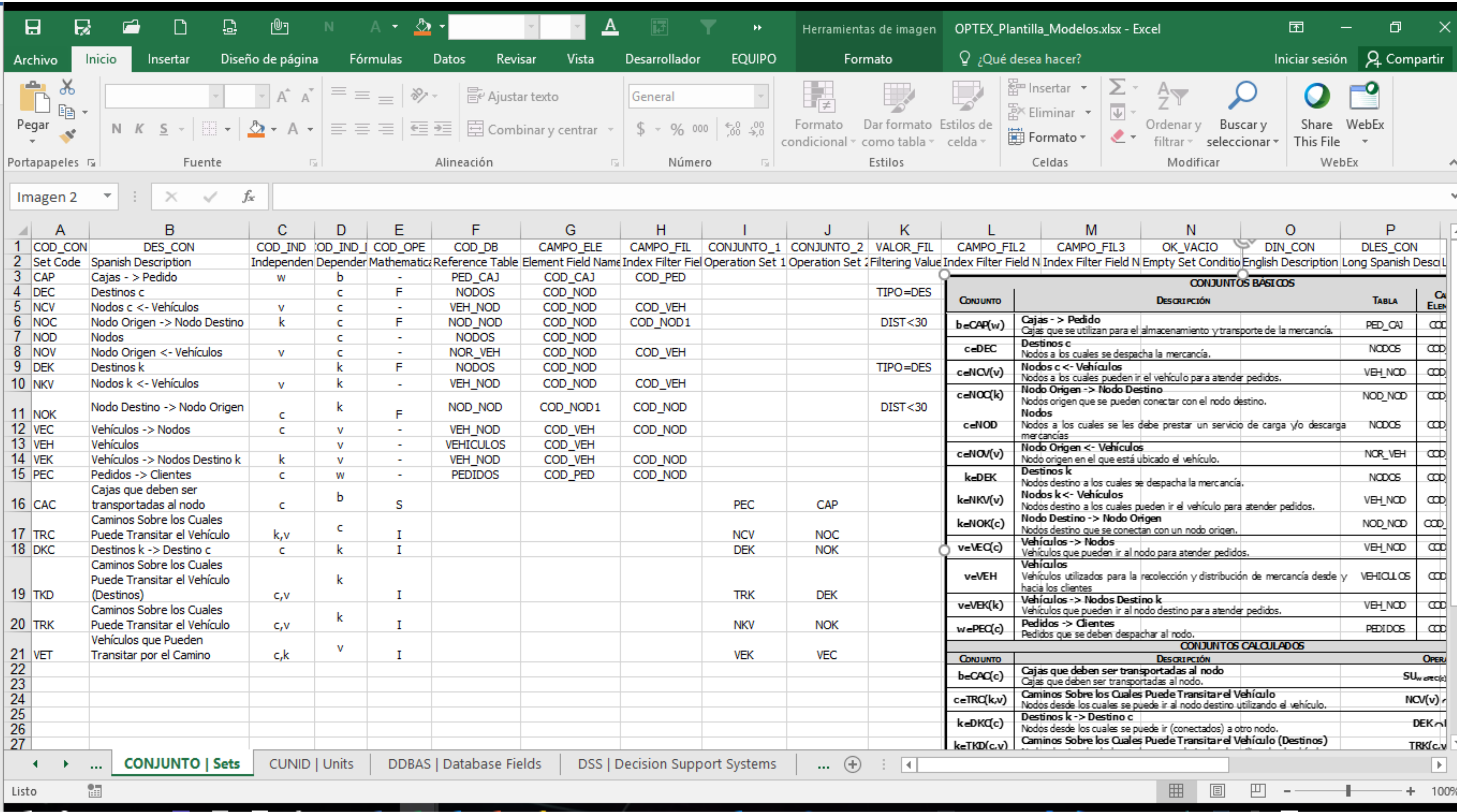
The definition of the sets implies filling three tables:

1. **The first (sheet CONJUNTO) which determines the general attributes of the sets**

2. **The second (sheet CON_IND) to define the independents indexes of the sets.**

SETS

1. **The first (sheet CONJUNTO) which determines the general attributes of the parameters. It includes many parameters like: tables, field, filters, type of operations, sets in operations, and independent index.**



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1	COD_CON	DES_CON	COD_IND	:OD_IND_I	COD_OPE	COD_DB	CAMPO_ELE	CAMPO_FIL	CONJUNTO_1	CONJUNTO_2	VALOR_FIL	CAMPO_FIL2	CAMPO_FIL3	OK_VACIO	DIN_CON	DLES_CON
2	Set Code	Spanish Description	Independen	Depender	Mathematic	Reference Table	Element Field Name	Index Filter Fiel	Operation Set 1	Operation Set 2	Filtering Value	Index Filter Field N	Index Filter Field N	Empty Set Conditio	English Description	Long Spanish Desc
3	CAP	Cajas - > Pedido	w	b	-	PED_CAJ	COD_CAJ	COD_PED								
4	DEC	Destinos c		c	F	NODOS	COD_NOD				TIPO=DES					
5	NCV	Nodos c <- Vehículos	v	c	-	VEH_NOD	COD_NOD	COD_VEH								
6	NOC	Nodo Origen -> Nodo Destino	k	c	F	NOD_NOD	COD_NOD	COD_NOD1			DIST<30					
7	NOD	Nodos		c	-	NODOS	COD_NOD									
8	NOV	Nodo Origen <- Vehículos	v	c	-	NOR_VEH	COD_NOD	COD_VEH								
9	DEK	Destinos k		k	F	NODOS	COD_NOD				TIPO=DES					
10	NKV	Nodos k <- Vehículos	v	k	-	VEH_NOD	COD_NOD	COD_VEH								
11	NOK	Nodo Destino -> Nodo Origen	c	k	F	NOD_NOD	COD_NOD1	COD_NOD			DIST<30					
12	VEC	Vehículos -> Nodos	c	v	-	VEH_NOD	COD_VEH	COD_NOD								
13	VEH	Vehículos		v	-	VEHICULOS	COD_VEH									
14	VEK	Vehículos -> Nodos Destino k	k	v	-	VEH_NOD	COD_VEH	COD_NOD								
15	PEC	Pedidos -> Clientes	c	w	-	PEDIDOS	COD_PED	COD_NOD								
16	CAC	Cajas que deben ser transportadas al nodo	c	b	S				PEC	CAP						
17	TRC	Caminos Sobre los Cuales Puede Transitar el Vehículo	k,v	c	I				NCV	NOC						
18	DKC	Destinos k -> Destino c	c	k	I				DEK	NOK						
19	TKD	Caminos Sobre los Cuales Puede Transitar el Vehículo (Destinos)	c,v	k	I				TRK	DEK						
20	TRK	Caminos Sobre los Cuales Puede Transitar el Vehículo	c,v	k	I				NKV	NOK						
21	VET	Vehículos que Pueden Transitar por el Camino	c,k	v	I				VEK	VEC						

CONJUNTOS BÁSICOS			
Conjunto	Descripción	Tabla	Ca Elem
beCAP(w)	Cajas - > Pedido Cajas que se utilizan para el almacenamiento y transporte de la mercancía.	PED_CAJ	COD
ceDEC	Destinos c Nodos a los cuales se despacha la mercancía.	NODOS	COD
ceNCV(v)	Nodos c <- Vehículos Nodos a los cuales pueden ir el vehículo para atender pedidos.	VEH_NOD	COD
ceNOC(k)	Nodo Origen -> Nodo Destino Nodos origen que se pueden conectar con el nodo destino.	NOD_NOD	COD
ceNOD	Nodos Nodos a los cuales se les debe prestar un servicio de carga y/o descarga mercancías	NODOS	COD
ceNOV(v)	Nodo Origen <- Vehículos Nodo origen en el que está ubicado el vehículo.	NOR_VEH	COD
keDEK	Destinos k Nodos destino a los cuales se despacha la mercancía.	NODOS	COD
keNKV(v)	Nodos k <- Vehículos Nodos destino a los cuales pueden ir el vehículo para atender pedidos.	VEH_NOD	COD
keNOK(c)	Nodo Destino -> Nodo Origen Nodos destino que se conectan con un nodo origen.	NOD_NOD	COD_
veVEC(c)	Vehículos -> Nodos Vehículos que pueden ir al nodo para atender pedidos.	VEH_NOD	COD
veVEH	Vehículos Vehículos utilizados para la recolección y distribución de mercancía desde y hacia los clientes	VEHICULOS	COD
veVEK(k)	Vehículos -> Nodos Destino k Vehículos que pueden ir al nodo destino para atender pedidos.	VEH_NOD	COD
wePEC(c)	Pedidos -> Clientes Pedidos que se deben despachar al nodo.	PEDIDOS	COD
CONJUNTOS CALCULADOS			
Conjunto	Descripción		Opera
beCAC(c)	Cajas que deben ser transportadas al nodo Cajas que deben ser transportadas al nodo.		SU
ceTRC(k,v)	Caminos Sobre los Cuales Puede Transitar el Vehículo Nodos desde los cuales se puede ir al nodo destino utilizando el vehículo.		NCV(v)
keDKC(c)	Destinos k -> Destino c Nodos desde los cuales se puede ir (conectados) a otro nodo.		DEK
keTKD(c,v)	Caminos Sobre los Cuales Puede Transitar el Vehículo (Destinos)		TRK(c,v

SETS

2. The second (sheet CON_IND) to define the independents indexes of the sets. The reference set (COD_CON1) may be used to reference when an empty validation is required.

PROBLEMS

Once all the structural elements have been defined then you must set models that you can/want to resolve, each one has associated with it a set of constraints that define it. Then general VRP presents two problems associated with the problem formulation, both belong to the MIP (Mixed Integer Programming) format.

Each problem is defined by the constraints that are included in the problem.

VRP PROBLEMS				
CODE/ID	**DESCRIPTION**	**PROBLEM FORMAT**	**RESTRICTION**	
			BASICS	**WEIGHT VOLUME**
VRP	Vehicle Routing (VRP)	**MIP**	**ENSA UTVE NOCL VCLI SANO**	
VRP2C	Vehicle Routing (VRP) - Weigth + Volume	**MIP**		**CAPP CAPV**

PROBLEMS

The problem definition involves filling four tables, the VRP only uses two of them.

1. **The first (sheet PROBLEMA) which determines the general attributes of the problem. In this table has been filled the role of the problem which is IN (integrated).**

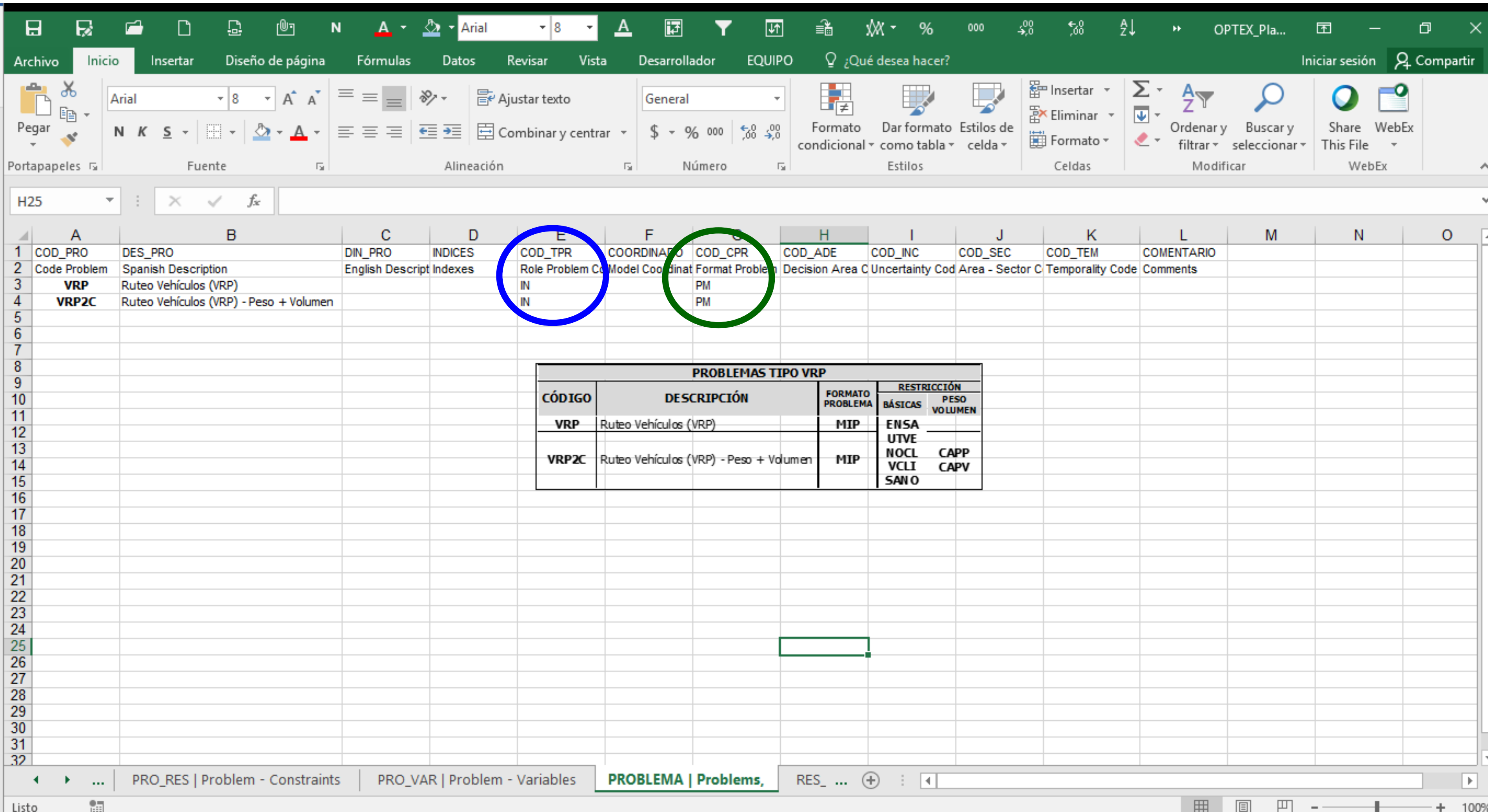
2. **The second (sheet PRO_RES) defines the restrictions that are included into the problem:**

<table>
<tr><th colspan="5">VRP PROBLEMS</th></tr>
<tr><th rowspan="2">CODE/ID</th><th rowspan="2">DESCRIPTION</th><th rowspan="2">PROBLEM FORMAT</th><th colspan="2">RESTRICTION</th></tr>
<tr><th>BASICS</th><th>WEIGHT VOLUME</th></tr>
<tr><td>VRP</td><td>Vehicle Routing (VRP)</td><td>MIP</td><td rowspan="2">ENSA
UTVE
NOCL
VCLI
SANO</td><td></td></tr>
<tr><td>VRP2C</td><td>Vehicle Routing (VRP) - Weigth + Volume</td><td>MIP</td><td>CAPP
CAPV</td></tr>
</table>

OPTΣX Mathematical Modeling System

PROBLEMS

1. **The first (sheet PROBLEMA) which determines the general attributes of the problem. In this table has been filled the role of the problem which is IN (integrated) and the format PM (Mixed Programming).**

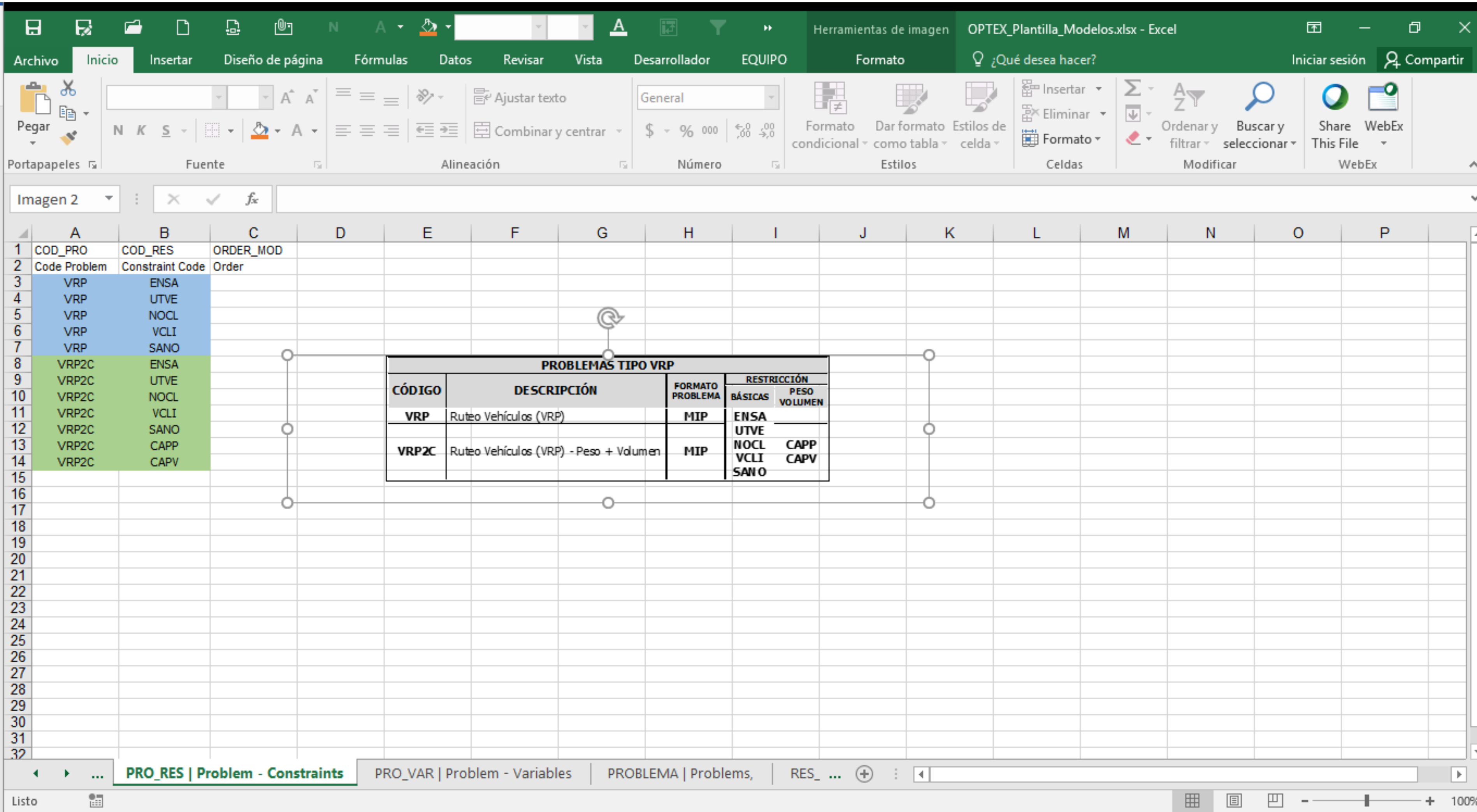


A	B	C	D	E	F	G	H	I	J	K	L
COD_PRO	DES_PRO	DIN_PRO	INDICES	COD_TPR	COORDINADO	COD_CPR	COD_ADE	COD_INC	COD_SEC	COD_TEM	COMENTARIO
Code Problem	Spanish Description	English Descript	Indexes	Role Problem C	Model Coordinat	Format Problem	Decision Area C	Uncertainty Cod	Area - Sector C	Temporality Code	Comments
VRP	Ruteo Vehículos (VRP)			IN		PM					
VRP2C	Ruteo Vehículos (VRP) - Peso + Volumen			IN		PM					

PROBLEMAS TIPO VRP				
CÓDIGO	DESCRIPCIÓN	FORMATO PROBLEMA	RESTRICCIÓN BÁSICAS	RESTRICCIÓN PESO VOLUMEN
VRP	Ruteo Vehículos (VRP)	MIP	ENSA	
VRP2C	Ruteo Vehículos (VRP) - Peso + Volumen	MIP	UTVE NOCL VCLI SANO	CAPP CAPV

PROBLEMS

2. **The second (sheet PRO_RES) defines the restrictions that are included into the problem. The field ORDER_MOD is optional and it is related whit the order of the constrains in the matrix of constraints; it may be useful in MIP models.**



MODELS

Finally, you must define the models considered in the application VRP, it implies to specify the problems that make up each model. In this case two models, one for each defined problem, are as shown in the following table.

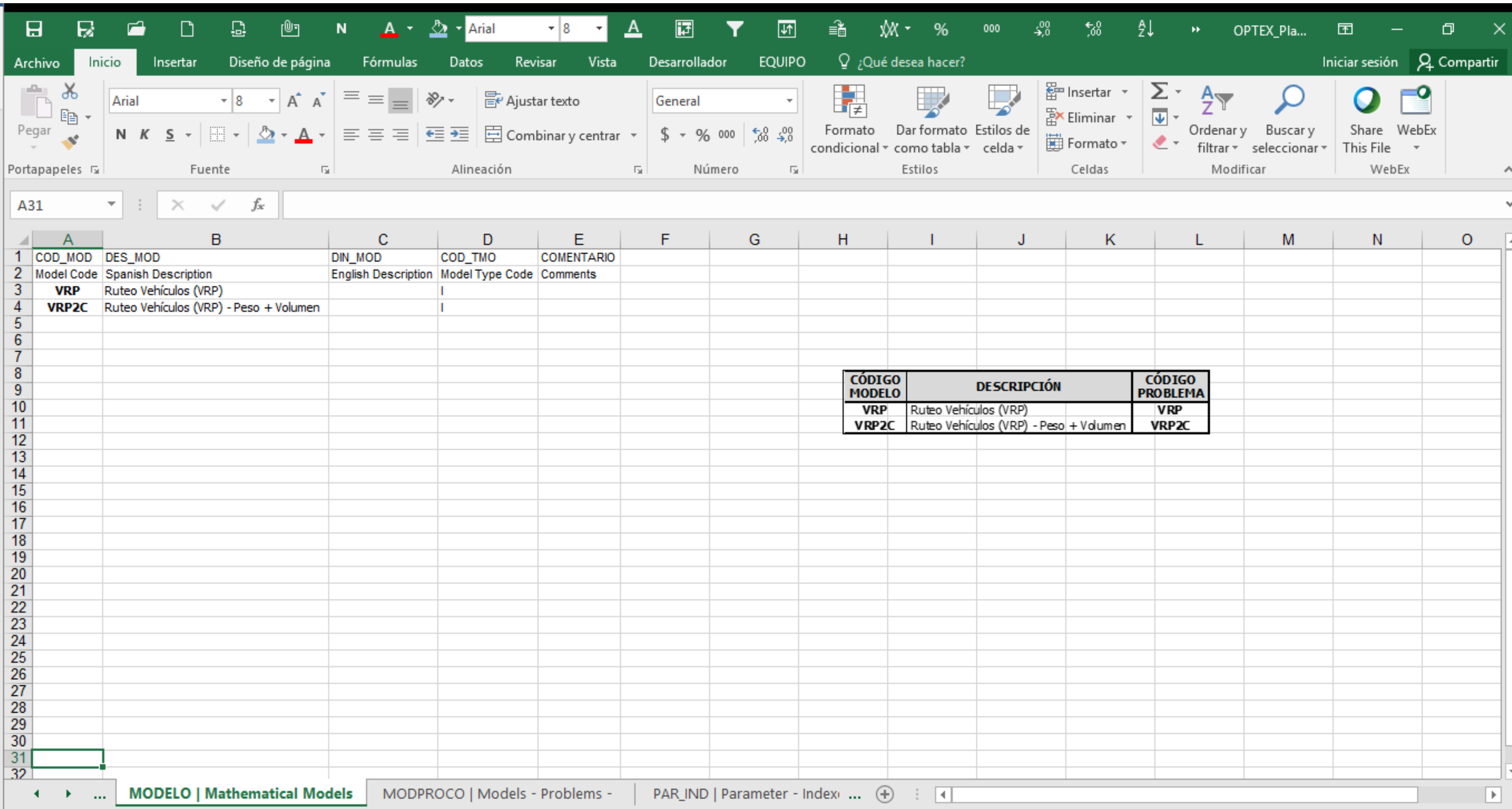
MODEL ID	DESCRIPTION	PROBLEM ID
VRP	Vehicle Routing (VRP)	**VRP**
VRP2C	Vehicle Routing (VRP) - Weigth + Volume	**VRP2C**

The definition of the models involves filling two tables:

1. **The first (MODEL) which determines the general attributes of each model.**
2. **The second (MOD_PRO) defines the problems that belong to the model.**

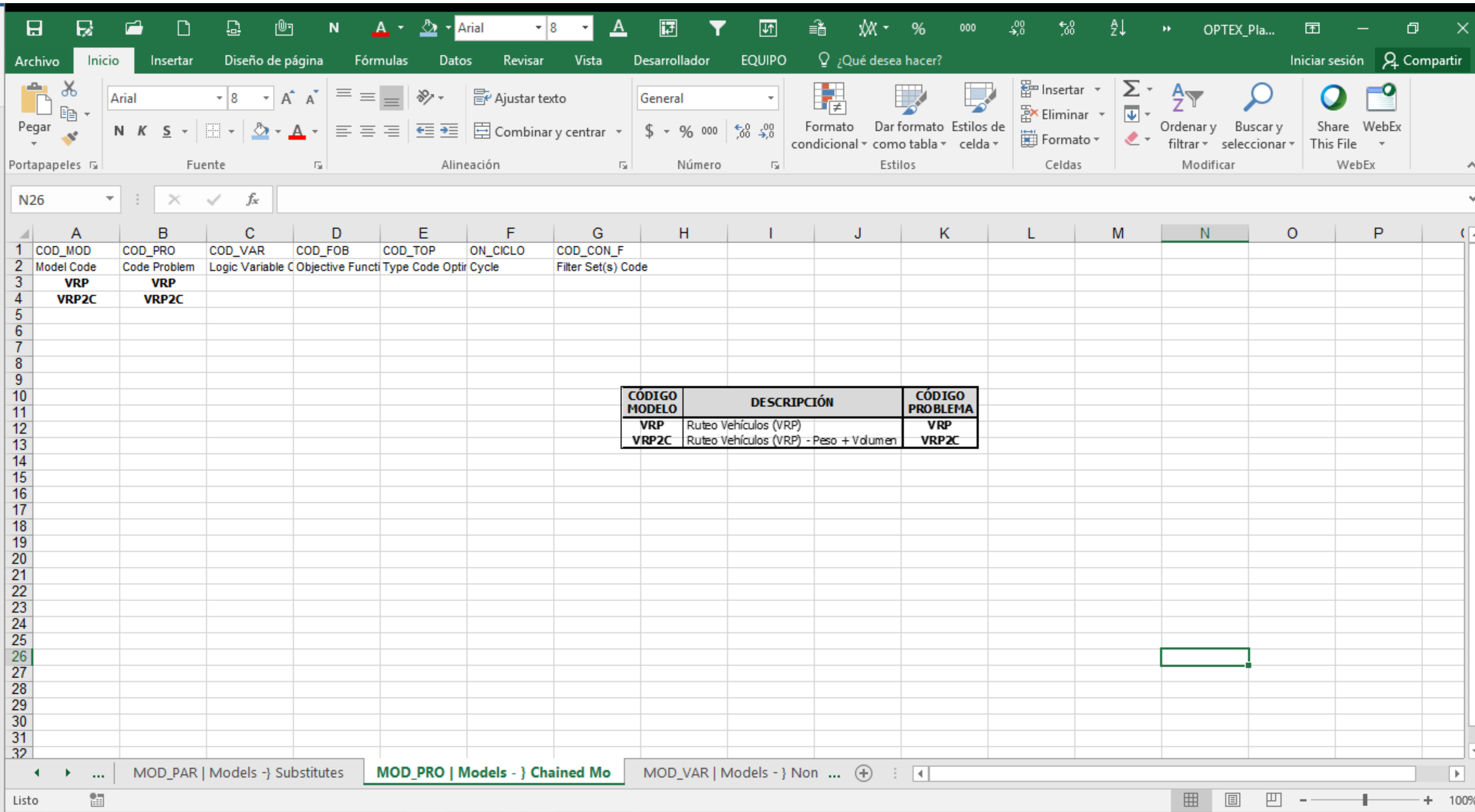
MODELS

1. **The first (sheet MODELO) which determines the general attributes of each model. In this table, the type of model has been filled as integrated (I), which implies that model corresponds to a unique mathematical problem.**



MODELS

2. The second (MOD_PRO) defines the problems that belong to the model.



Now, You have seen the way to formulate mathematical models in EXCEL that can be solved the using OPTEX-EXCEL-MMS template.

Two alternatives you have:

1. **To check the model. It may be using the OPTEX-WEB interface**

2. **To build the database of the VRP models (IDIS)**

SUGGESTION:
If the reader considers it convenient, DOA suggests that load the template of models from the tables included in this document.

For this you can download the WORD version of the tables included in this manual from the URL:

http://www.doanalytics.net/Documents/Manual-Tutorial-OPTEX-Implementacion-Modelo-VRP-Tables.rar

DO Analytics

"the computer-based mathematical modeling is the greatest invention of all times"

Herbert Simon
First Winner of Nobel Prize in Economics (1978)

"for his pioneering research into the decision-making process within economic organizations"